

Worked Solutions: An Introduction to Surds

Part 1: Square roots on the calculator

Q1

The completed table is:

Square root	Calculator answer (4 d.p.)	Is it a surd?
$\sqrt{16}$	4	No
$\sqrt{2}$	1.4142	Yes
$\sqrt{49}$	7	No
$\sqrt{3}$	1.7320	Yes
$\sqrt{10}$	3.1622	Yes

A surd is a square root that cannot be written exactly as a whole number or a fraction. So $\sqrt{16}$ and $\sqrt{49}$ are not surds; $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{10}$ are surds.

Part 2: Surds that are equal

Q2

(a) Using the calculator and truncating to 4 decimal places:

1. $2\sqrt{2} = 2.8284$
2. $\sqrt{12} = 3.4641$
3. $\sqrt{6} = 2.4494$
4. $\sqrt{18} = 4.2426$
5. $\sqrt{20} = 4.4721$
6. $2\sqrt{3} = 3.4641$
7. $\sqrt{8} = 2.8284$
8. $3\sqrt{5} = 6.7082$
9. $2\sqrt{5} = 4.4721$
10. $3\sqrt{2} = 4.2426$

(b) The four equal pairs are:

- $2\sqrt{2} = \sqrt{8}$

- $\sqrt{12} = 2\sqrt{3}$
- $\sqrt{18} = 3\sqrt{2}$
- $\sqrt{20} = 2\sqrt{5}$

The unpaired numbers are $\sqrt{6}$ and $3\sqrt{5}$.

Explanation: for example, $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$, so $\sqrt{12}$ is equal to $2\sqrt{3}$.

Q3

(a)

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= \sqrt{4} \times \sqrt{3} \\ &= 2\sqrt{3}.\end{aligned}$$

(b)

$$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5\sqrt{2}.\end{aligned}$$

(c) $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}.$

(d) $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}.$

(e) $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}.$

(f) $\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}.$

(g) $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}.$

(h) $\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}.$

Part 3: Adding and subtracting surds

Q4

(a) $2x + 4x = (2 + 4)x = 6x.$

(b) $5y - 2y = (5 - 2)y = 3y.$

(c) $a + 3a = 1a + 3a = 4a.$

(d) $6m - m = 6m - 1m = 5m.$

(e) $4x + 2y + x = (4 + 1)x + 2y = 5x + 2y.$

(f) $x + y$ cannot be simplified because x and y are not like terms.

Q5

(a) $3\sqrt{5} + 2\sqrt{5} = (3 + 2)\sqrt{5} = 5\sqrt{5}.$

- (b) $9\sqrt{7} - 4\sqrt{7} = (9 - 4)\sqrt{7} = 5\sqrt{7}$.
- (c) $4\sqrt{2} + \sqrt{2} = 4\sqrt{2} + 1\sqrt{2} = 5\sqrt{2}$.
- (d) $3\sqrt{6} + 5\sqrt{6} = 8\sqrt{6}$.
- (e) $8\sqrt{11} - 3\sqrt{11} = 5\sqrt{11}$.
- (f) $6\sqrt{5} - \sqrt{5} = 6\sqrt{5} - 1\sqrt{5} = 5\sqrt{5}$.
- (g) $2\sqrt{3} + 3\sqrt{3} + \sqrt{3} = (2 + 3 + 1)\sqrt{3} = 6\sqrt{3}$.
- (h) $5\sqrt{2} + 3\sqrt{7} + 2\sqrt{2} = (5 + 2)\sqrt{2} + 3\sqrt{7} = 7\sqrt{2} + 3\sqrt{7}$.
- (i) $\sqrt{2} + \sqrt{3}$ cannot be simplified because $\sqrt{2}$ and $\sqrt{3}$ are not like surds. It is not $\sqrt{5}$.
- (j) $4\sqrt{6} - 4\sqrt{6} = 0$.

Q6

- (a) The rectangle has two sides of length $3\sqrt{2}$ and two sides of length $\sqrt{2}$. So

$$\text{Perimeter} = 3\sqrt{2} + \sqrt{2} + 3\sqrt{2} + \sqrt{2} = (3 + 1 + 3 + 1)\sqrt{2} = 8\sqrt{2} \text{ cm.}$$

- (b) The triangle has sides $2\sqrt{5}$, $3\sqrt{5}$ and $4\sqrt{5}$. So

$$\text{Perimeter} = 2\sqrt{5} + 3\sqrt{5} + 4\sqrt{5} = (2 + 3 + 4)\sqrt{5} = 9\sqrt{5} \text{ cm.}$$

Q7

- (a) $\sqrt{8} + \sqrt{2} = \sqrt{4 \times 2} + \sqrt{2} = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$.
- (b) $\sqrt{12} + \sqrt{3} = \sqrt{4 \times 3} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$.
- (c) $\sqrt{18} - \sqrt{2} = \sqrt{9 \times 2} - \sqrt{2} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$.
- (d) $\sqrt{45} + \sqrt{20} = \sqrt{9 \times 5} + \sqrt{4 \times 5} = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$.

Part 4: Multiplying and dividing surds

Q8

- (a) $\sqrt{2} \times \sqrt{7} = \sqrt{2 \times 7} = \sqrt{14}$.
- (b) $3\sqrt{2} \times 5\sqrt{7} = (3 \times 5)\sqrt{2 \times 7} = 15\sqrt{14}$.
- (c) $\sqrt{3} \times \sqrt{7} = \sqrt{21}$.
- (d) $\sqrt{6} \times \sqrt{6} = 6$.
- (e) $\sqrt{2} \times \sqrt{18} = \sqrt{36} = 6$.
- (f) $\sqrt{5} \times \sqrt{20} = \sqrt{100} = 10$.
- (g) $2\sqrt{5} \times 3\sqrt{2} = (2 \times 3)\sqrt{5 \times 2} = 6\sqrt{10}$.
- (h) $4\sqrt{3} \times 2\sqrt{7} = (4 \times 2)\sqrt{3 \times 7} = 8\sqrt{21}$.
- (i) $5\sqrt{2} \times \sqrt{3} = 5\sqrt{6}$.
- (j) $3\sqrt{5} \times 3\sqrt{5} = (3 \times 3)(\sqrt{5} \times \sqrt{5}) = 9 \times 5 = 45$.

Q9

- (a) $\sqrt{21} \div \sqrt{7} = \sqrt{21 \div 7} = \sqrt{3}$.
- (b) $10\sqrt{15} \div 5\sqrt{5} = (10 \div 5)\sqrt{15 \div 5} = 2\sqrt{3}$.
- (c) $\sqrt{30} \div \sqrt{6} = \sqrt{30 \div 6} = \sqrt{5}$.
- (d) $\frac{\sqrt{22}}{\sqrt{2}} = \sqrt{\frac{22}{2}} = \sqrt{11}$.
- (e) $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$.
- (f) $\sqrt{63} \div \sqrt{7} = \sqrt{63 \div 7} = \sqrt{9} = 3$.
- (g) $8\sqrt{6} \div 4\sqrt{2} = (8 \div 4)\sqrt{6 \div 2} = 2\sqrt{3}$.
- (h) $12\sqrt{10} \div 3\sqrt{2} = (12 \div 3)\sqrt{10 \div 2} = 4\sqrt{5}$.

Q10

- (a) Area = $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4 \text{ cm}^2$.
- (b) Area = $5\sqrt{2} \times 2\sqrt{3} = (5 \times 2)\sqrt{2 \times 3} = 10\sqrt{6} \text{ cm}^2$.
- (c) Length = $\frac{\text{Area}}{\text{width}} = \frac{\sqrt{35}}{\sqrt{5}} = \sqrt{\frac{35}{5}} = \sqrt{7} \text{ cm}$.

Part 5: Many ways to make a surd

Q11

For each part, one valid completion is given.

(a) Middle $5\sqrt{2}$:

- Addition: $\sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$.
- Subtraction: $8\sqrt{2} - 3\sqrt{2} = 5\sqrt{2}$.
- Multiplication: $\sqrt{25} \times \sqrt{2} = 5\sqrt{2}$.
- Division: $10\sqrt{6} \div 2\sqrt{3} = 5\sqrt{2}$.
- Single square root: $\sqrt{50} = 5\sqrt{2}$.

(b) Middle $3\sqrt{5}$:

- Addition: $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$ (given).
- Subtraction: $4\sqrt{5} - \sqrt{5} = 3\sqrt{5}$.
- Multiplication: $\sqrt{9} \times \sqrt{5} = 3\sqrt{5}$.
- Division: $6\sqrt{10} \div 2\sqrt{2} = 3\sqrt{5}$.
- Single square root: $\sqrt{45} = 3\sqrt{5}$ (given).

(c) Middle $6\sqrt{2}$:

- Addition: $2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$.
- Subtraction: $7\sqrt{2} - \sqrt{2} = 6\sqrt{2}$.
- Multiplication: $\sqrt{36} \times \sqrt{2} = 6\sqrt{2}$.
- Division: $12\sqrt{6} \div 2\sqrt{3} = 6\sqrt{2}$.
- Single square root: $\sqrt{72} = 6\sqrt{2}$.

(d) Middle $2\sqrt{7}$:

- Addition: $\sqrt{7} + \sqrt{7} = 2\sqrt{7}$.
- Subtraction: $5\sqrt{7} - 3\sqrt{7} = 2\sqrt{7}$.
- Multiplication: $\sqrt{4} \times \sqrt{7} = 2\sqrt{7}$.
- Division: $6\sqrt{14} \div 3\sqrt{2} = 2\sqrt{7}$.
- Single square root: $\sqrt{28} = 2\sqrt{7}$.

(e) One possible choice: middle $3\sqrt{2}$.

- Addition: $\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$.
- Subtraction: $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$.
- Multiplication: $\sqrt{9} \times \sqrt{2} = 3\sqrt{2}$.
- Division: $9\sqrt{6} \div 3\sqrt{3} = 3\sqrt{2}$.
- Single square root: $\sqrt{18} = 3\sqrt{2}$.

In every case, check the box by simplifying the calculation to see that it gives the middle surd.