

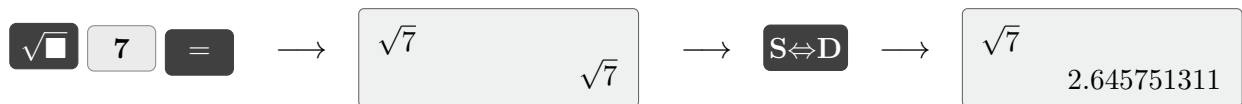
An Introduction to Surds

A **surd** is a square root that cannot be written exactly as a whole number or a fraction, such as $\sqrt{2}$.

Part 1: Square roots on the calculator

The **square root** of a number is the number that gives it when it is multiplied by itself. For example $\sqrt{9} = 3$, because $3 \times 3 = 9$.

To find a square root on the calculator we press the square root button, type the number and press equals. Many calculators give the answer as a surd. To see the answer as a decimal, press the $S \Leftrightarrow D$ button.



The calculator shows $\sqrt{7} = 2.645751311$, but the decimal does not stop at the edge of the screen. It carries on forever, so we write $\sqrt{7}$ instead.

Q1. Use the square root button to complete the table. When the answer is not a whole number, truncate to the first four decimal places.

Square root	Calculator answer	Is it a surd?
$\sqrt{16}$	4	No
$\sqrt{2}$		
$\sqrt{49}$		
$\sqrt{3}$		
$\sqrt{10}$		

Part 2: Surds that are equal

$3\sqrt{2}$ means $3 \times \sqrt{2}$, which is three lots of $\sqrt{2}$, in the same way that $3x$ means $3 \times x$ in algebra. To find $3\sqrt{2}$ we press $\boxed{3}$ $\boxed{\sqrt{\square}}$ $\boxed{2}$ $\boxed{=}$ on the calculator.

Q2. (a) Use your calculator to find each answer, truncated to four decimal places.

1. $2\sqrt{2} =$ _____

6. $2\sqrt{3} =$ _____

2. $\sqrt{12} =$ _____

7. $\sqrt{8} =$ _____

3. $\sqrt{6} =$ _____

8. $3\sqrt{5} =$ _____

4. $\sqrt{18} =$ _____

9. $2\sqrt{5} =$ _____

5. $\sqrt{20} =$ _____

10. $3\sqrt{2} =$ _____

(b) Four pairs of numbers in part (a) are equal to each other. Write down the four pairs, in surd form. Two of the numbers do not have a partner.

_____ = _____

_____ = _____

_____ = _____

_____ = _____

The pairs in Q2 are equal because we can factor out a square number. For example $8 = 4 \times 2$, and $\sqrt{4} = 2$, so $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$. Writing $\sqrt{8}$ as $2\sqrt{2}$ is called **simplifying** the surd.

Formulas: simplifying surds

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b} \quad \text{for example} \quad \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

In words: split the number under the root into a square number multiplied by another number. The square root of the square number is written in front of the surd.

Square numbers: 4, 9, 16, 25, 36, 49, 64, 81, 100

Worked example: simplify $\sqrt{18}$

Step 1 — split the number using a square number. $\sqrt{18} = \sqrt{9 \times 2}$

Step 2 — split the root. $\sqrt{18} = \sqrt{9} \times \sqrt{2}$

Step 3 — take the square root of the square number. $\sqrt{18} = 3 \times \sqrt{2} = 3\sqrt{2}$

Q3. Fill in the gaps in parts (a) and (b), then simplify parts (c) to (h) in the same way.

(a) Simplify $\sqrt{12}$.

Step 1 — split the number using a square number. $\sqrt{12} = \sqrt{\boxed{} \times 3}$

Step 2 — split the root. $\sqrt{12} = \sqrt{\boxed{}} \times \sqrt{3}$

Step 3 — take the square root of the square number. $\sqrt{12} = \underline{\hspace{1cm}} \times \sqrt{3} = \underline{\hspace{1cm}} \sqrt{3}$

(b) Simplify $\sqrt{50}$.

Step 1 — split the number using a square number. $\sqrt{50} = \sqrt{\boxed{} \times \boxed{}}$

Step 2 — split the root. $\sqrt{50} =$ _____

Step 3 — take the square root of the square number. $\sqrt{50} =$ _____

(c) $\sqrt{24}$

(e) $\sqrt{45}$

(g) $\sqrt{75}$

(d) $\sqrt{28}$

(f) $\sqrt{63}$

(h) $\sqrt{98}$

Part 3: Adding and subtracting surds

Q4. Warm-up. Simplify each expression by collecting like terms. If an expression cannot be simplified, write it as it is.

(a) $2x + 4x =$ _____

(c) $a + 3a =$ _____

(e) $4x + 2y + x =$ _____

(b) $5y - 2y =$ _____

(d) $6m - m =$ _____

(f) $x + y =$ _____

Like terms have the same letter, and we collect them by adding or subtracting the numbers in front. Surds are collected in the same way: a surd such as $\sqrt{3}$ is collected just like the letter x . Surds with the same number under the root, such as $2\sqrt{3}$ and $4\sqrt{3}$, are called **like surds**. A surd with no number in front has 1 lot, so $\sqrt{3} = 1\sqrt{3}$, just as $x = 1x$.

Formulas: adding and subtracting like surds

$$a\sqrt{n} + b\sqrt{n} = (a + b)\sqrt{n} \quad \text{for example} \quad 2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

$$a\sqrt{n} - b\sqrt{n} = (a - b)\sqrt{n} \quad \text{for example} \quad 9\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$$

In words: add or subtract the numbers in front, and keep the surd the same.

Only like surds can be collected. Just as $x + y$ cannot be simplified, $\sqrt{2} + \sqrt{3}$ cannot be simplified. It is **not** $\sqrt{5}$: on the calculator $\sqrt{2} + \sqrt{3} = 3.146264\dots$, but $\sqrt{5} = 2.236067\dots$

Worked example: collecting like surds

Each line of surds is collected in the same way as the algebra beside it.

Algebra

$$2x + 4x = (2 + 4)x = 6x$$

$$7y - 3y = (7 - 3)y = 4y$$

$$4x + 2y + x = 5x + 2y$$

Surds

$$2\sqrt{3} + 4\sqrt{3} = (2 + 4)\sqrt{3} = 6\sqrt{3}$$

$$7\sqrt{2} - 3\sqrt{2} = (7 - 3)\sqrt{2} = 4\sqrt{2}$$

$$4\sqrt{2} + 2\sqrt{3} + \sqrt{2} = 5\sqrt{2} + 2\sqrt{3}$$

Q5. Fill in the gaps in parts (a) and (b), then simplify parts (c) to (j) using the formula. Some of them cannot be simplified.

(a) $3\sqrt{5} + 2\sqrt{5} = (\underline{\quad} + \underline{\quad})\sqrt{5} = \underline{\quad}\sqrt{5}$

(b) $9\sqrt{7} - 4\sqrt{7} = (\underline{\quad} - \underline{\quad})\sqrt{7} = \underline{\quad}$

(c) $4\sqrt{2} + \sqrt{2}$

(f) $6\sqrt{5} - \sqrt{5}$

(i) $\sqrt{2} + \sqrt{3}$

(d) $3\sqrt{6} + 5\sqrt{6}$

(g) $2\sqrt{3} + 3\sqrt{3} + \sqrt{3}$

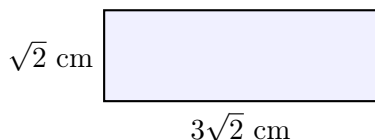
(j) $4\sqrt{6} - 4\sqrt{6}$

(e) $8\sqrt{11} - 3\sqrt{11}$

(h) $5\sqrt{2} + 3\sqrt{7} + 2\sqrt{2}$

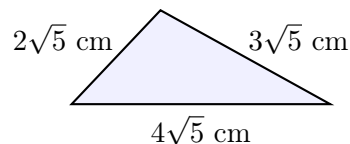
Q6. The **perimeter** of a shape is the total length of its sides.

(a) The rectangle has sides of length $3\sqrt{2}$ cm and $\sqrt{2}$ cm. Fill in the gaps to find its perimeter.



Perimeter = $3\sqrt{2} + \sqrt{2} + \underline{\quad} + \underline{\quad}$
 = $\underline{\quad}$ cm

(b) The triangle has sides of length $2\sqrt{5}$ cm, $3\sqrt{5}$ cm and $4\sqrt{5}$ cm. Work out its perimeter.



Perimeter = $\underline{\quad}$ cm

Q7. Harder. In each of these the first surd is not a like surd until it has been simplified. Simplify it using the formula from Part 2, then collect like surds.

(a) Fill in the gaps: $\sqrt{8} + \sqrt{2} = \underline{\quad}\sqrt{2} + \sqrt{2} = \underline{\quad}\sqrt{2}$

(b) $\sqrt{12} + \sqrt{3}$

(c) $\sqrt{18} - \sqrt{2}$

(d) $\sqrt{45} + \sqrt{20}$

Part 4: Multiplying and dividing surds

Part 2 showed that $\sqrt{18} = \sqrt{9} \times \sqrt{2}$. Read the other way round, this says $\sqrt{9} \times \sqrt{2} = \sqrt{18}$: when we multiply two square roots, we multiply the numbers under the roots. Dividing works in the same way. Remember that ab means $a \times b$.

Formulas: multiplying and dividing surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

for example $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

$$\sqrt{a} \div \sqrt{b} = \sqrt{a \div b} \quad \text{or} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

for example $\sqrt{15} \div \sqrt{3} = \sqrt{5}$

$$\sqrt{a} \times \sqrt{a} = a$$

for example $\sqrt{7} \times \sqrt{7} = 7$

$$m\sqrt{a} \times n\sqrt{b} = mn\sqrt{ab}$$

for example $2\sqrt{3} \times 4\sqrt{5} = 8\sqrt{15}$

$$m\sqrt{a} \div n\sqrt{b} = (m \div n)\sqrt{a \div b}$$

for example $8\sqrt{6} \div 2\sqrt{3} = 4\sqrt{2}$

In words: multiply or divide the numbers under the roots. Numbers in front are multiplied or divided separately.

Worked example: multiplying surds

$$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$$

$$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$$

the answer can be a whole number

$$2\sqrt{3} \times 4\sqrt{5} = (2 \times 4)\sqrt{3 \times 5} = 8\sqrt{15}$$

the numbers in front are multiplied separately

Q8. Fill in the gaps in parts (a) and (b), then work out parts (c) to (j) using the formulas. Some of the answers are whole numbers.

(a) $\sqrt{2} \times \sqrt{7} = \sqrt{\boxed{} \times \boxed{}} = \sqrt{\boxed{}}$

(b) $3\sqrt{2} \times 5\sqrt{7} = (\underline{} \times \underline{})\sqrt{\boxed{} \times \boxed{}} = \underline{}$

(c) $\sqrt{3} \times \sqrt{7}$

(e) $\sqrt{2} \times \sqrt{18}$

(g) $2\sqrt{5} \times 3\sqrt{2}$

(i) $5\sqrt{2} \times \sqrt{3}$

(d) $\sqrt{6} \times \sqrt{6}$

(f) $\sqrt{5} \times \sqrt{20}$

(h) $4\sqrt{3} \times 2\sqrt{7}$

(j) $3\sqrt{5} \times 3\sqrt{5}$

Worked example: dividing surds

$$\sqrt{15} \div \sqrt{3} = \sqrt{15 \div 3} = \sqrt{5}$$

$$\sqrt{50} \div \sqrt{2} = \sqrt{50 \div 2} = \sqrt{25} = 5 \quad \text{the answer can be a whole number}$$

$$8\sqrt{6} \div 2\sqrt{3} = (8 \div 2)\sqrt{6 \div 3} = 4\sqrt{2} \quad \text{the numbers in front are divided separately}$$

Q9. Fill in the gaps in parts (a) and (b), then work out parts (c) to (h) using the formulas. Some of the answers are whole numbers.

(a) $\sqrt{21} \div \sqrt{7} = \sqrt{\boxed{} \div \boxed{}} = \sqrt{\boxed{}}$

(b) $10\sqrt{15} \div 5\sqrt{5} = (___ \div ___) \sqrt{\boxed{} \div \boxed{}} = ______$

(c) $\sqrt{30} \div \sqrt{6}$

(e) $\frac{\sqrt{12}}{\sqrt{3}}$

(g) $8\sqrt{6} \div 4\sqrt{2}$

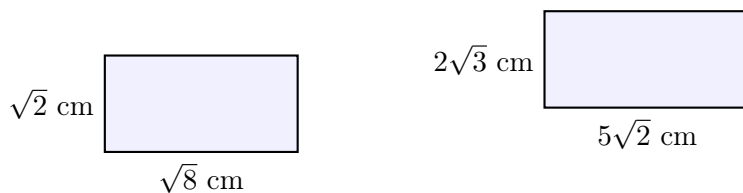
(d) $\frac{\sqrt{22}}{\sqrt{2}}$

(f) $\sqrt{63} \div \sqrt{7}$

(h) $12\sqrt{10} \div 3\sqrt{2}$

Q10. The shapes are not drawn to scale. The area of a rectangle is its length multiplied by its width.

(a) Fill in the gaps to find the area. (b) Work out the area.

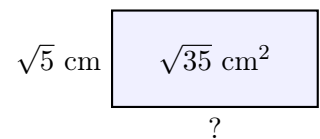


Area = $\sqrt{2} \times \sqrt{8}$

= $\sqrt{\boxed{}} = ______ \text{ cm}^2$

Area = $______ \text{ cm}^2$

(c) The area is $\sqrt{35} \text{ cm}^2$. Work out the length.



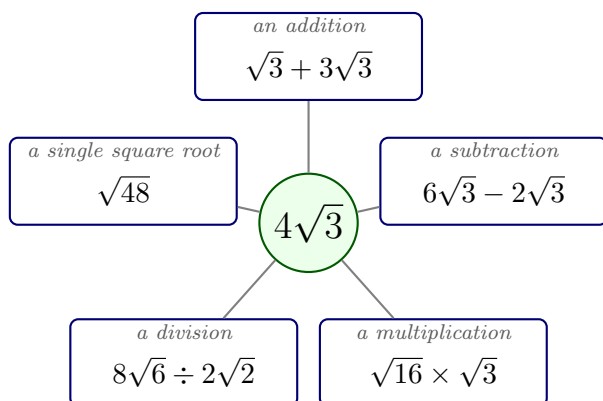
Length = $______ \text{ cm}$

Part 5: Many ways to make a surd

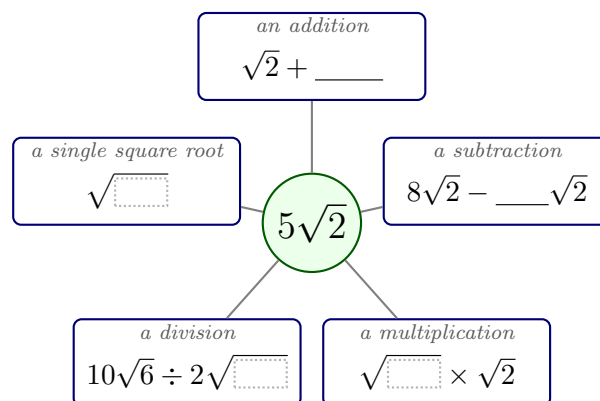
Each box around a surd holds a calculation whose answer is that surd. Use the formulas from this sheet, and check each box on your calculator. The first diagram is done for you.

Q11. Complete each diagram.

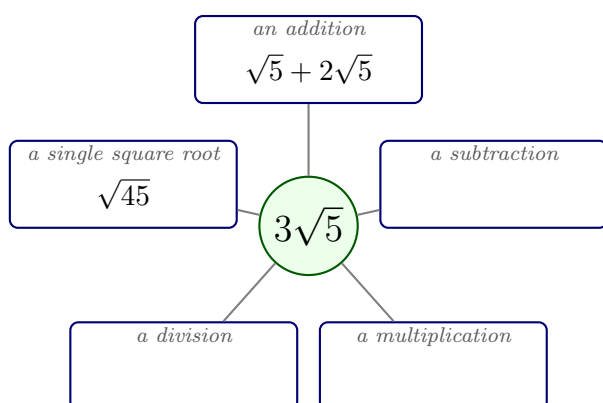
Example



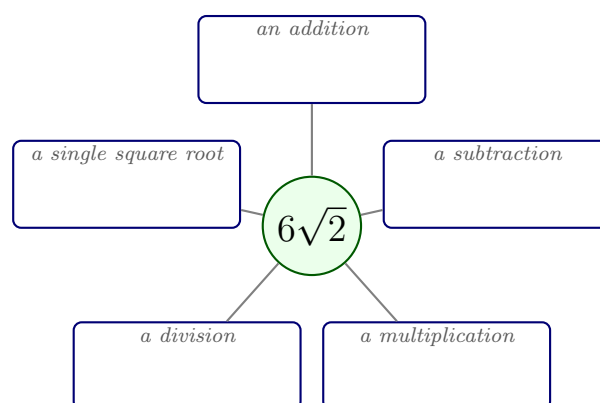
(a) Fill in the gaps.



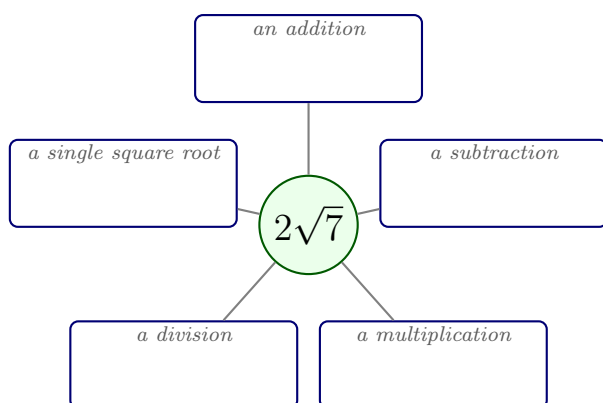
(b) Two boxes are done. Fill in the other three.



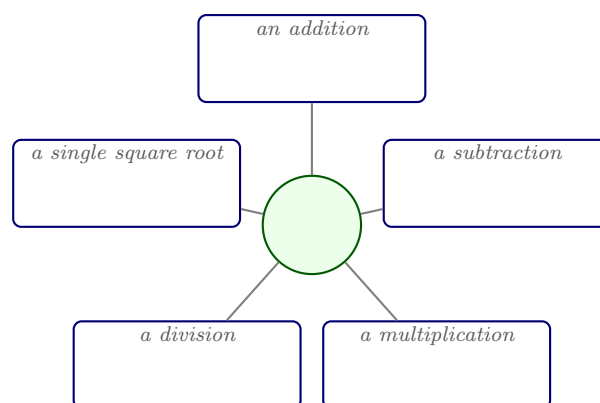
(c) Fill in all five boxes.



(d) Fill in all five boxes.



(e) Choose your own surd for the middle.



Answers

Q1. $\sqrt{16} = 4$, not a surd. $\sqrt{2} = 1.414213\dots$, a surd. $\sqrt{49} = 7$, not a surd. $\sqrt{3} = 1.732050\dots$, a surd. $\sqrt{10} = 3.162277\dots$, a surd.

Q2. (a) 1. 2.828427 2. 3.464101 3. 2.449489
4. 4.242640 5. 4.472135 6. 3.464101 7. 2.828427
8. 6.708203 9. 4.472135 10. 4.242640 (b) $2\sqrt{2} = \sqrt{8}$, $\sqrt{12} = 2\sqrt{3}$, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{20} = 2\sqrt{5}$. The numbers $\sqrt{6}$ and $3\sqrt{5}$ do not have a partner.

Q3. (a) $\sqrt{4 \times 3}$; $\sqrt{4} \times \sqrt{3}$; $2 \times \sqrt{3} = 2\sqrt{3}$
(b) $\sqrt{25 \times 2}$; $\sqrt{25} \times \sqrt{2}$; $5 \times \sqrt{2} = 5\sqrt{2}$ (c)
 $\sqrt{4 \times 6} = 2\sqrt{6}$ (d) $\sqrt{4 \times 7} = 2\sqrt{7}$ (e) $\sqrt{9 \times 5} = 3\sqrt{5}$ (f) $\sqrt{9 \times 7} = 3\sqrt{7}$ (g) $\sqrt{25 \times 3} = 5\sqrt{3}$ (h)
 $\sqrt{49 \times 2} = 7\sqrt{2}$

Q4. (a) $6x$ (b) $3y$ (c) $4a$ (d) $5m$ (e) $5x + 2y$
(f) cannot

Q5. (a) $(3 + 2)\sqrt{5} = 5\sqrt{5}$ (b) $(9 - 4)\sqrt{7} = 5\sqrt{7}$
(c) $5\sqrt{2}$ (d) $8\sqrt{6}$ (e) $5\sqrt{11}$ (f) $5\sqrt{5}$ (g) $6\sqrt{3}$
(h) $7\sqrt{2} + 3\sqrt{7}$ (i) cannot: $\sqrt{2}$ and $\sqrt{3}$ are not like surds (j) 0

Q6. (a) $3\sqrt{2} + \sqrt{2} + 3\sqrt{2} + \sqrt{2} = 8\sqrt{2}$ cm (b)
 $2\sqrt{5} + 3\sqrt{5} + 4\sqrt{5} = 9\sqrt{5}$ cm

Q7. (a) $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$ (b) $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$
(c) $3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$ (d) $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Q8. (a) $\sqrt{2 \times 7} = \sqrt{14}$ (b) $(3 \times 5)\sqrt{2 \times 7} = 15\sqrt{14}$
(c) $\sqrt{21}$ (d) 6 (e) $\sqrt{36} = 6$ (f) $\sqrt{100} = 10$ (g)
 $6\sqrt{10}$ (h) $8\sqrt{21}$ (i) $5\sqrt{6}$ (j) $9 \times 5 = 45$

Q9. (a) $\sqrt{21 \div 7} = \sqrt{3}$ (b) $(10 \div 5)\sqrt{15 \div 5} = 2\sqrt{3}$
(c) $\sqrt{5}$ (d) $\sqrt{11}$ (e) $\sqrt{4} = 2$ (f) $\sqrt{9} = 3$ (g)
 $2\sqrt{3}$ (h) $4\sqrt{5}$

Q10. (a) $\sqrt{16} = 4$ cm² (b) $2\sqrt{3} \times 5\sqrt{2} = 10\sqrt{6}$
cm² (c) $\sqrt{35} \div \sqrt{5} = \sqrt{7}$ cm

Q11. (a) $\sqrt{2} + 4\sqrt{2}$; $8\sqrt{2} - 3\sqrt{2}$; $\sqrt{25} \times \sqrt{2}$;
 $10\sqrt{6} \div 2\sqrt{3}$; $\sqrt{50}$. Parts (b) to (e) have many
answers. For example: (b) $4\sqrt{5} - \sqrt{5}$; $\sqrt{9} \times \sqrt{5}$;
 $6\sqrt{10} \div 2\sqrt{2}$. (c) $2\sqrt{2} + 4\sqrt{2}$; $7\sqrt{2} - \sqrt{2}$; $\sqrt{36} \times \sqrt{2}$;
 $12\sqrt{6} \div 2\sqrt{3}$; $\sqrt{72}$. (d) $\sqrt{7} + \sqrt{7}$; $5\sqrt{7} - 3\sqrt{7}$;
 $\sqrt{4} \times \sqrt{7}$; $6\sqrt{14} \div 3\sqrt{2}$; $\sqrt{28}$. (e) Any surd, with
five calculations that give it.