

Inequalities

Revision questions: worked solutions

Reading and drawing inequalities on number lines

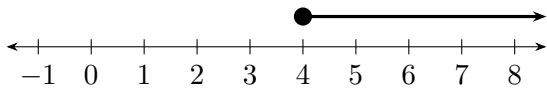
Remember: Open circle: not included. Filled circle: included.

Q1. (a) Open circle at 1, arrow to the right: $x > 1$

(b) Filled circle at -2 , arrow to the left: $x \leq -2$

(c) Filled circle at -3 and open circle at 2: $-3 \leq x < 2$

Q2. (a) $\geq$ includes 4, so a filled circle at 4, with an arrow to the right.



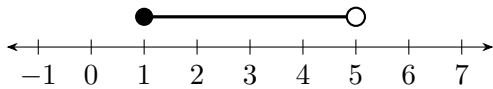
(b) -1 is not included (open circle), and 5 is included (filled circle).



Q3. (a) -1 is not included, and 5 is included: 0, 1, 2, 3, 4, 5

(b) Neither end is included: $-4, -3, -2$

Q4. “At least 1” includes 1, and “less than 5” does not include 5: $1 \leq t < 5$



Solving inequalities

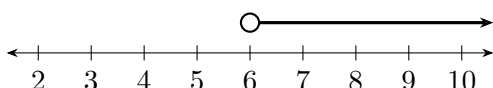
Remember: Solve it like an equation, but keep the sign.

Q5.

$x + 7 > 12$	$4x \leq 28$	$2x - 3 < 11$	$\frac{x}{5} + 2 \geq 4$
$-7 \quad -7$	$\div 4 \quad \div 4$	$+3 \quad +3$	$-2 \quad -2$
$x > 5$	$x \leq 7$	$2x < 14$	$\frac{x}{5} \geq 2$
		$\div 2 \quad \div 2$	$\times 5 \quad \times 5$
		$x < 7$	$x \geq 10$

Q6. Expand the bracket first: $3x - 6 > 12$.

$$\begin{aligned}
 3x - 6 &> 12 \\
 +6 \quad +6 \\
 3x &> 18 \\
 \div 3 \quad \div 3 \\
 x &> 6
 \end{aligned}$$



Solving inequalities with the unknown on both sides

Remember: Take away the smaller x -term from both sides first.

Q7.

$$\begin{array}{rcl} 5x + 2 > 3x + 10 & 8x - 1 \leq 5x + 11 & 2x + 8 < x + 13 \\ -3x & -5x & -x \\ 2x + 2 > 10 & 3x - 1 \leq 11 & x + 8 < 13 \\ -2 & +1 & -8 \\ 2x > 8 & 3x \leq 12 & x < 5 \\ \div 2 & \div 3 & \\ x > 4 & x \leq 4 & \end{array}$$

In (c) we expand the bracket first: $2(x + 4) = 2x + 8$.

Solving double inequalities

Remember: Whatever you do to one part, do to all three parts.

Q8.

$$\begin{array}{rcl} 5 < x + 3 \leq 9 & -6 \leq 3x < 9 & 1 < 2x - 5 < 9 \\ -3 & \div 3 & +5 \\ 2 < x \leq 6 & -2 \leq x < 3 & 6 < 2x < 14 \\ & & \div 2 \\ & & 3 < x < 7 \end{array}$$

(a) 3, 4, 5, 6 (b) -2, -1, 0, 1, 2 (c) 4, 5, 6

Q9.

$$\begin{array}{rcl} -7 < 4n + 1 \leq 13 & & \\ -1 & -1 & -1 \\ -8 < 4n \leq 12 & & \\ \div 4 & \div 4 & \div 4 \\ -2 < n \leq 3 & & \end{array}$$

-2 is not included, so the smallest integer is -1. 3 is included, so the largest is 3.

Constructing inequalities

Remember: At most $\leq$, at least $\geq$, more than $>$, less than $<$.

Q10. "More than" is $>$.

$$\begin{array}{rcl} 3n - 4 > 20 & & \\ +4 & +4 & \\ 3n > 24 & & \\ \div 3 & \div 3 & \\ n > 8 & & \end{array}$$

Q11. The cost is $3 + 2m$, and Ella can spend at most £19, so $\leq$.

$$3 + 2m \leq 19$$

$$\begin{array}{r} -3 \quad -3 \\ 2m \leq 16 \end{array}$$

$$\begin{array}{r} \div 2 \quad \div 2 \\ m \leq 8 \end{array}$$

The greatest whole number of miles is 8.

Q12. Perimeter $= x + (x + 6) + x + (x + 6) = 4x + 12$. "At least" is $\geq$.

$$4x + 12 \geq 40$$

$$\begin{array}{r} -12 \quad -12 \\ 4x \geq 28 \end{array}$$

$$\begin{array}{r} \div 4 \quad \div 4 \\ x \geq 7 \end{array}$$

The smallest possible value of x is 7.

Inequalities on graphs

Remember: Dashed: not on it. Solid: on it. Test a point.

Q13. (a) The line is $x = 3$, solid, and the region is to the left: $x \leq 3$

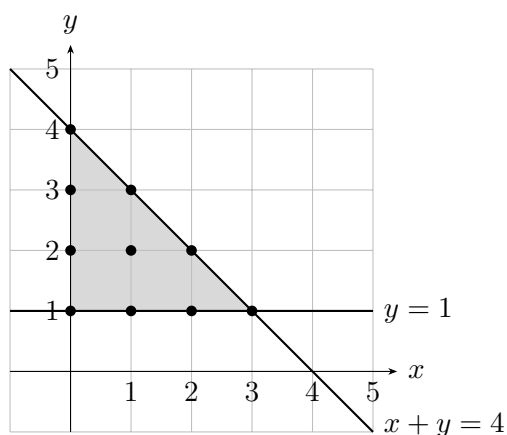
(b) The line is $y = -1$, dashed, and the region is above: $y > -1$

(c) The line is $y = x + 1$, solid, and the region is above. Test $(0, 3)$: $3 \geq 0 + 1$ ✓, so $y \geq x + 1$

Q14. $y = 1$ is solid and R is above it: $y \geq 1$. $x = 4$ is dashed and R is to its left: $x < 4$. $y = x$ is solid and R is below it: $y \leq x$.

Test a point in R, $(3, 2)$: $2 \geq 1$ ✓, $3 < 4$ ✓, $2 \leq 3$ ✓

Q15. The three lines are $x = 0$ (the y -axis), $y = 1$ and $x + y = 4$, all solid. The region is the triangle with corners $(0, 1)$, $(3, 1)$ and $(0, 4)$.



Counting the points, including the edges: $4 + 3 + 2 + 1 = 10$ points.

Dividing by a negative number

Remember: Times or divide by a negative? Flip the sign!

Q16.

$$\begin{array}{lll} -2x > 8 & -5x \leq -15 & 7 - 3x < 1 \\ \div(-2) \quad \div(-2) & \div(-5) \quad \div(-5) & \begin{array}{cc} -7 & -7 \\ -3x < -6 \end{array} \\ x < -4 & x \geq 3 & \div(-3) \quad \div(-3) \\ & & x > 2 \quad \text{flipped} \end{array}$$

Q17. With $x = 4$: $10 - 2 \times 4 = 2$, and 2 is not more than 4. So $x = 4$ does not work, and Jo's answer $x > 3$ is wrong.

$$\begin{array}{l} 10 - 2x > 4 \\ \begin{array}{cc} -10 & -10 \\ -2x > -6 \end{array} \\ \div(-2) \quad \div(-2) \\ x < 3 \quad \text{flipped} \end{array}$$

Jo divided by -2 without flipping the sign.

Problem solving

Q18. $3x - 1 > 5$ gives $3x > 6$, so $x > 2$. $2x + 3 \leq 15$ gives $2x \leq 12$, so $x \leq 6$.

Both are true when $2 < x \leq 6$: the integers are 3, 4, 5, 6.

Q19. The inequality must include -2 and 1 , but not -3 or 2 . For example $-3 < x < 2$, or $-2 \leq x \leq 1$.

Q20. In pence, plan A costs $1000 + 5m$ and plan B costs $1800 + m$. Plan B is cheaper when

$$\begin{array}{l} 1800 + m < 1000 + 5m \\ \begin{array}{cc} -m & -m \\ 1800 < 1000 + 4m \end{array} \\ \begin{array}{cc} -1000 & -1000 \\ 800 < 4m \end{array} \\ \begin{array}{cc} \div 4 & \div 4 \\ 200 < m \end{array} \end{array}$$

Plan B is cheaper for more than 200 minutes of calls. (At exactly 200 minutes both plans cost £20.)