

Fractions, decimals and percentages

Revision questions: worked solutions

Simplifying fractions

Remember: What you do to the numerator, do to the denominator.

Q1. 18 of the 24 parts are shaded. The HCF of 18 and 24 is 6, so $\div$ top and bottom by 6:

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

Q2. (a) HCF of 12 and 16 is 4: $\frac{12}{16} = \frac{3}{4}$ (b) HCF of 30 and 45 is 15: $\frac{30}{45} = \frac{2}{3}$ (c) HCF of 84 and 120 is 12: $\frac{84}{120} = \frac{7}{10}$

Ordering fractions

Remember: Common denominator, then compare the numerators.

Q3. (a) Convert to denominator 24: $\frac{3}{4} = \frac{18}{24}$, $\frac{5}{8} = \frac{15}{24}$, $\frac{7}{12} = \frac{14}{24}$, so the order is $\frac{7}{12}$, $\frac{5}{8}$, $\frac{3}{4}$.

(b) Convert to denominator 20: $\frac{4}{5} = \frac{16}{20}$, $\frac{7}{10} = \frac{14}{20}$, $\frac{3}{4} = \frac{15}{20}$, $\frac{13}{20}$, so the order is $\frac{13}{20}$, $\frac{7}{10}$, $\frac{3}{4}$, $\frac{4}{5}$.

Q4. Convert to denominator 40: $\frac{3}{8} = \frac{15}{40}$ and $\frac{2}{5} = \frac{16}{40}$. $\frac{16}{40}$ is bigger, so Sam eats more.

Mixed numbers and improper fractions

Remember: Whole times denominator, add the numerator.

Q5. Each bar is one whole, split into quarters. Two whole bars are shaded, and 3 quarters of the third bar.

(a) $2\frac{3}{4}$ (b) $\frac{2 \times 4 + 3}{4} = \frac{11}{4}$ (11 quarters are shaded)

Q6. (a) $5\frac{2}{3} = \frac{5 \times 3 + 2}{3} = \frac{17}{3}$ (b) $31 \div 6 = 5$ remainder 1, so $\frac{31}{6} = 5\frac{1}{6}$

Adding and subtracting fractions

Remember: Common denominator. Never add the denominators!

Q7. (a) $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$ (b) $\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{13}{28}$

(c) $\frac{5}{6} + \frac{3}{8} = \frac{20}{24} + \frac{9}{24} = \frac{29}{24} = 1\frac{5}{24}$

Q8. Jack spends $\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$ of his money, so he has $1 - \frac{13}{20} = \frac{7}{20}$ left.

One number as a percentage of another

Remember: Part over whole, times 100.

Q9. (a) $\frac{12}{50} \times 100 = 24\%$ (b) $\frac{27}{60} \times 100 = 45\%$

(c) Same units first: 2 m = 200 cm, so $\frac{35}{200} \times 100 = 17.5\%$

Q10. Priya: $\frac{34}{40} \times 100 = 85\%$. Leo: $\frac{41}{50} \times 100 = 82\%$. Priya did better.

Percentages without a calculator

Remember: 10%: $\div 10$. 1%: $\div 100$. 50%: halve. Then build up.

Q11. (a) $470 \div 10 = 47$ (b) $86 \div 2 = 43$ (c) $650 \div 100 = 6.5$

(d) 10% of 360 = 36 and 5% = 18, so 15% = 36 + 18 = 54

Q12. 10% of £45 = £4.50, so 20% = £9. The sale price is $45 - 9 = £36$.

Converting between fractions, decimals and percentages

Remember: Top $\div$ bottom gives the decimal; $\times 100$ gives the %.

Q13. 36 of the 100 squares are shaded.

$$\frac{36}{100} = \frac{9}{25} \quad 0.36 \quad 36\%$$

($\div$ top and bottom by 4 to simplify)

Q14.	Fraction	Decimal	Percentage	
	$\frac{3}{5}$	0.6	60%	$3 \div 5 = 0.6$, then $\times 100$
	$\frac{9}{20}$	0.45	45%	$0.45 = \frac{45}{100} = \frac{9}{20}$ ($\div$ top and bottom by 5)
	$\frac{1}{25}$	0.04	4%	$4\% = \frac{4}{100} = \frac{1}{25}$, and $4 \div 100 = 0.04$

Ordering fractions, decimals and percentages

Remember: Turn everything into decimals, then compare them.

Q15. (a) $0.4 = 0.400$, $\frac{3}{8} = 0.375$, $41\% = 0.410$, so the order is $\frac{3}{8}$, 0.4, 41%.

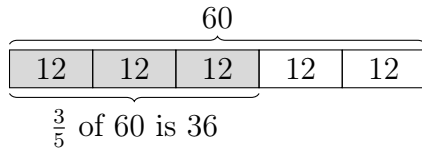
(b) $\frac{1}{3} = 0.333\dots$, $0.3 = 0.300$, $33\% = 0.330$, so the order is 0.3, 33%, $\frac{1}{3}$.

Q16. As decimals: Shop A $\frac{1}{3} = 0.333\dots$, Shop B 30% = 0.3, Shop C 0.35. The biggest is 0.35, so Shop C gives the biggest discount.

Fractions of amounts

Remember: Divide by the denominator, times by the numerator.

Q17. $60 \div 5 = 12$ in each part, and 3 parts are shaded: $12 \times 3 = 36$.



Q18. (a) $64 \div 8 = 8$, $8 \times 3 = 24$ (b) Calculator: $7 \div 9 \times 504 = 392$

Q19. $840 \div 7 = 120$, and $120 \times 3 = 360$ pupils have school dinners, so $840 - 360 = 480$ do not.

Percentages of amounts

Remember: Per cent means 'out of 100'; 'of' means multiply by.

Q20. (a) 10% of $140 = 14$, $5\% = 7$, so $35\% = 14 + 14 + 14 + 7 = 49$

(b) 50% of $64 = 32$, $25\% = 16$, so $12.5\% = 8$ (halving each time)

Q21. (a) $0.23 \times 85 = 19.55$ (b) $0.045 \times 2400 = 108$, so £108

Percentage change

Remember: Change over original, times 100.

Q22. (a) Change = $62 - 50 = 12$: $\frac{12}{50} \times 100 = 24\%$ increase

(b) Change = $80 - 68 = 12$: $\frac{12}{80} \times 100 = 15\%$ decrease

Q23. Change = $12\,000 - 9000 = 3000$: $\frac{3000}{12\,000} \times 100 = 25\%$ decrease

Simple interest

Remember: Same interest every year: find one year, multiply by years.

Q24. 2% of £500 = £10 interest every year.

Year	0	1	2	3	4
Balance (£)	500	510	520	530	540

Q25. 5% of £800 = £40 a year. Amy needs $1000 - 800 = £200$ of interest, and $200 \div 40 = 5$, so it takes 5 years.

Calculating with mixed numbers and negatives

Remember: Mixed $\rightarrow$ improper first. Dividing? Keep, change, flip. (KFC)

Q26. (a) $1\frac{2}{3} + 2\frac{1}{4} = \frac{5}{3} + \frac{9}{4} = \frac{20}{12} + \frac{27}{12} = \frac{47}{12} = 3\frac{11}{12}$

(b) $4\frac{1}{5} - 1\frac{1}{2} = \frac{21}{5} - \frac{3}{2} = \frac{42}{10} - \frac{15}{10} = \frac{27}{10} = 2\frac{7}{10}$

(c) $1\frac{1}{2} \times 2\frac{2}{3} = \frac{3}{2} \times \frac{8}{3} = \frac{24}{6} = 4$

Q27. (a) $3\frac{3}{4} \div 1\frac{1}{4} = \frac{15}{4} \div \frac{5}{4} = \frac{15}{4} \times \frac{4}{5} = \frac{60}{20} = 3$

(b) $-\frac{2}{3} \times \frac{9}{10} = -\frac{18}{30} = -\frac{3}{5}$

(c) $-1\frac{1}{4} \div \left(-\frac{5}{8}\right) = -\frac{5}{4} \times \left(-\frac{8}{5}\right) = \frac{40}{20} = 2$ (negative $\times$ negative = positive)

Word problems with mixed numbers

Remember: Find the numbers, pick the operation, include units.

Q28. Cut off: subtract.

$$5\frac{1}{4} - 2\frac{2}{3} = \frac{21}{4} - \frac{8}{3} = \frac{63}{12} - \frac{32}{12} = \frac{31}{12} = 2\frac{7}{12} \text{ m}$$

Q29. How many fit into: divide.

$$2\frac{3}{4} \div \frac{1}{3} = \frac{11}{4} \times \frac{3}{1} = \frac{33}{4} = 8\frac{1}{4}$$

Only full glasses count, so 8 glasses.

Q30. Area = length $\times$ width

$$6\frac{1}{2} \times 3\frac{1}{3} = \frac{13}{2} \times \frac{10}{3} = \frac{130}{6} = \frac{65}{3} = 21\frac{2}{3} \text{ m}^2$$

Recurring decimals to fractions

Remember: Line up the repeats, then subtract them away. Solve algebra.

Q31.

$$\begin{array}{rcl} \text{Let } n = 0.444\dots & & \\ 10n = 4.444\dots & n = \frac{4}{9} & \\ \text{subtract: } 9n = 4 & & \end{array}$$

Q32. (a) Let $n = 0.3636\dots$ Two digits repeat, so $\times 100$:

$$\begin{array}{rcl} 100n = 36.3636\dots & & \\ n = 0.3636\dots & n = \frac{36}{99} = \frac{4}{11} & \\ \text{subtract: } 99n = 36 & & \end{array}$$

(b) Let $n = 0.8333\dots$. The 8 does not repeat, so use $100n$ and $10n$ to line up the repeats:

$$\begin{array}{rcl} 100n & = & 83.333\dots \\ 10n & = & 8.333\dots \\ \text{subtract:} & 90n & = 75 \end{array} \qquad n = \frac{75}{90} = \frac{5}{6}$$

Percentage increase and decrease with multipliers

Remember: 100% plus or minus the change, then $\div 100$.

Q33. (a) $100\% + 8\% = 108\% \rightarrow 1.08$ (b) $100\% - 30\% = 70\% \rightarrow 0.7$ (c) $102.5\% \rightarrow 1.025$

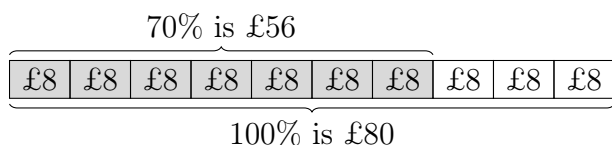
Q34. (a) Up 35% $\rightarrow \times 1.35$: $64 \times 1.35 = 86.4$, so £86.40

(b) Down 12% $\rightarrow \times 0.88$: $1250 \times 0.88 = 1100$

Finding the original amount

Remember: Going backwards? Divide by the multiplier!

Q35. 30% off $\rightarrow \times 0.7$: $0.7 \times \text{original} = 56 \rightarrow \text{original} = 56 \div 0.7 = £80$



Check with the bar: 70% is £56, so 10% is £8 and 100% is £80.

Q36. Up 10% $\rightarrow \times 1.1$: original = $77 \div 1.1 = £70$

Q37. Up 20% $\rightarrow \times 1.2$: original = $54 \div 1.2 = £45$

Problem solving

Q38. Shop A: $\frac{1}{3}$ of 90 = 30, so the coat costs £60.

Shop B: $90 \times 0.7 = 63$, then $63 \times 0.95 = 59.85$, so the coat costs £59.85.

Shop B is cheaper, by $60 - 59.85 = £0.15$, which is 15p.

Q39. Kim has worked forwards from £280, but the 30% was taken off the original price, not off £280. Going backwards, we divide by the multiplier: 30% off $\rightarrow \times 0.7$, so

$$\text{original} = 280 \div 0.7 = £400$$

Check: 30% of £400 is £120, and $400 - 120 = 280$. Kim's £364 does not work: 30% off £364 is £254.80.

Q40. $\frac{3}{5} = 0.6$ and $\frac{7}{10} = 0.7$, so any fraction whose decimal is between 0.6 and 0.7 will do. For example

$$\frac{13}{20} = 0.65 \qquad \frac{31}{50} = 0.62 \qquad \frac{2}{3} = 0.666\dots$$

Another way: with denominator 40, $\frac{3}{5} = \frac{24}{40}$ and $\frac{7}{10} = \frac{28}{40}$, so $\frac{25}{40}$, $\frac{26}{40}$ and $\frac{27}{40}$ are all in between.