

Simplifying Ratios

1. $14 : 21$

Solution. Divide both parts by 7:

$$14 : 21 = 2 : 3.$$

2. $24 : 18 : 42$

Solution. Divide all three parts by 6:

$$24 : 18 : 42 = 4 : 3 : 7.$$

3. $1.5 : 4.5$

Solution. Multiply both parts by 2, then divide by 3:

$$1.5 : 4.5 = 3 : 9 = 1 : 3.$$

4. $\frac{2}{3} : \frac{4}{5}$

Solution. Multiply both parts by the LCM of 3 and 5, which is 15:

$$\frac{2}{3} : \frac{4}{5} = \left(\frac{2}{3} \times 15\right) : \left(\frac{4}{5} \times 15\right) = 10 : 12 = 5 : 6.$$

5. White : blue : yellow paint = $600 : 250 : 150$.

Solution. Divide all parts by their HCF, 50:

$$600 : 250 : 150 = 12 : 5 : 3.$$

Sharing into Ratios

1. Share 24 sweets in the ratio $3 : 1$.

Solution. Total parts = $3 + 1 = 4$. One part = $24 \div 4 = 6$ sweets.

$$3 \times 6 = 18, \quad 1 \times 6 = 6.$$

So the two children receive 18 sweets and 6 sweets.

2. Divide £80 in the ratio $3 : 2$.

Solution. Total parts = $3 + 2 = 5$. One part = $80 \div 5 = 16$.

$$3 \times 16 = 48, \quad 2 \times 16 = 32.$$

So the two people receive £48 and £32.

3. Divide 180 students in the ratio $2 : 3 : 4$.

Solution. Total parts = $2 + 3 + 4 = 9$. One part = $180 \div 9 = 20$.

$$2 \times 20 = 40, \quad 3 \times 20 = 60, \quad 4 \times 20 = 80.$$

The groups contain 40, 60, and 80 students.

4. A 2.4 m piece of wood is cut in the ratio 1 : 2 : 5.

Solution. Total parts = $1 + 2 + 5 = 8$. One part = $2.4 \div 8 = 0.3$ m.

$$1 \times 0.3 = 0.3, \quad 2 \times 0.3 = 0.6, \quad 5 \times 0.3 = 1.5.$$

The lengths are 0.3 m, 0.6 m, and 1.5 m.

5. Marbles in the ratio red : blue : green = 2 : 5 : 3, total 150 marbles.

Solution. Total parts = $2 + 5 + 3 = 10$. One part = $150 \div 10 = 15$.

$$\text{red} = 2 \times 15 = 30, \quad \text{blue} = 5 \times 15 = 75, \quad \text{green} = 3 \times 15 = 45.$$

There are 30 red, 75 blue, and 45 green marbles.

Recipe Questions

1. Recipe for 4 servings, need 10 servings.

Solution. Scale factor = $\frac{10}{4} = \frac{5}{2}$.

- Minced beef: $400 \times \frac{5}{2} = 1000$ g = 1 kg.
- Onions: $2 \times \frac{5}{2} = 5$ onions, or $300 \times \frac{5}{2} = 750$ g.
- Chopped tomatoes: $400 \times \frac{5}{2} = 1000$ g = 1 kg.
- Stock: $150 \times \frac{5}{2} = 375$ ml.

2. Cookie recipe makes 15 cookies, need 35 cookies.

Solution. Scale factor = $\frac{35}{15} = \frac{7}{3}$.

- Flour: $225 \times \frac{7}{3} = 525$ g.
- Butter: $150 \times \frac{7}{3} = 350$ g.
- Sugar: $100 \times \frac{7}{3} = \frac{700}{3} = 233\frac{1}{3}$ g.
- Egg: $50 \times \frac{7}{3} = \frac{350}{3} = 116\frac{2}{3}$ g of beaten egg, i.e. $2\frac{1}{3}$ eggs.

3. Dressing recipe for 2 portions, need 7 portions.

Solution. Scale factor = $\frac{7}{2}$.

- Olive oil: $60 \times \frac{7}{2} = 210$ ml.
- Vinegar: $30 \times \frac{7}{2} = 105$ ml.
- Mustard: $10 \times \frac{7}{2} = 35$ g.
- Honey: $5 \times \frac{7}{2} = 17.5$ g.

4. Lemonade ratio lemon juice : sugar : water = 2 : 1 : 8, total 3.3 litres.

Solution. Total parts = $2 + 1 + 8 = 11$.

$$\text{one part} = \frac{3.3}{11} = 0.3 \text{ litres.}$$

$$\text{lemon juice} = 2 \times 0.3 = 0.6 \text{ L,} \quad \text{sugar} = 1 \times 0.3 = 0.3 \text{ L,} \quad \text{water} = 8 \times 0.3 = 2.4 \text{ L.}$$

5. Curry recipe for 4 people.

(a) For 11 people, scale factor = $\frac{11}{4}$.

Chicken: $400 \times \frac{11}{4} = 1100$ g = 1.1 kg.

Rice: $200 \times \frac{11}{4} = 550$ g.

Coconut milk: $300 \times \frac{11}{4} = 825$ ml.

Spices: multiply each of the 6 spice quantities by $\frac{11}{4}$. If counting spice items, this is $6 \times \frac{11}{4} = 16.5$ spice-units.

(b) With 600 g of chicken:

$$\frac{600}{400} = 1.5$$

so the recipe can be scaled by 1.5. This serves

$$4 \times 1.5 = 6$$

people.

(c) Using the 600 g chicken, scale factor = $\frac{600}{400} = \frac{3}{2}$.

Chicken: 600 g.

Rice: $200 \times \frac{3}{2} = 300$ g.

Coconut milk: $300 \times \frac{3}{2} = 450$ ml.

Spices: multiply each spice quantity by $\frac{3}{2}$; equivalently, $6 \times \frac{3}{2} = 9$ spice-units.

Direct Proportion

1. 4 pencils cost 96p.

Solution. One pencil costs $96 \div 4 = 24$ p.

$$7 \times 24 = 168 \text{ p} = \text{£}1.68.$$

2. A car travels 180 km in 3 hours.

Solution. Speed = $180 \div 3 = 60$ km/h.

$$60 \times 7 = 420 \text{ km.}$$

3. 2 litres of paint covers 15 m².

Solution.

$$\frac{90}{15} = 6$$

so 6 times as much paint is needed.

$$2 \times 6 = 12 \text{ litres.}$$

4. A 2.5 m rope costs £1.35.

Solution. Unit price = $1.35 \div 2.5 = 0.54$ pounds per metre.

$$3.24 \div 0.54 = 6 \text{ m.}$$

Bob bought 6 metres of rope.

5. A 1.2 m metal bar has mass 4.5 kg.

Solution. 80 cm = 0.8 m.

Unit mass = $4.5 \div 1.2 = 3.75$ kg/m.

$$3.75 \times 0.8 = 3 \text{ kg.}$$

The bar has mass 3 kg.

Best Buys

1. 200 g coffee for £5.40 or 350 g for £8.75.

Solution. Use pence per gram.

$$540 \div 200 = 2.7 \text{ p/g,} \quad 875 \div 350 = 2.5 \text{ p/g.}$$

The 350 g pack is cheaper per gram.

2. Pencils: 12 for £4.80 or 30 for £11.00.

Solution.

$$480 \div 12 = 40 \text{ p per pencil,} \quad 1100 \div 30 \approx 36.7 \text{ p per pencil.}$$

The pack of 30 offers better value per pencil.

3. Milk: 2 litres for £2.85 or 5 litres for £6.50.

Solution.

$$2.85 \div 2 = 1.425 \text{ pounds per litre,} \quad 6.50 \div 5 = 1.30 \text{ pounds per litre.}$$

The 5 litre bottle is the better buy.

4. Rice options: A: 8 kg for £24.80, B: 15 kg for £44.25, C: 25 kg for £72.50.

Solution.

$$A : 24.80 \div 8 = 3.10 \text{ pounds per kg,}$$

$$B : 44.25 \div 15 = 2.95 \text{ pounds per kg,}$$

$$C : 72.50 \div 25 = 2.90 \text{ pounds per kg.}$$

Option C offers the best value per kilogram.

5. Car rental: X: £45 per day, unlimited mileage. Y: £35 per day plus £0.15 per mile.
Plan: 250 miles over 3 days.

Solution.

$$\text{Company X cost} = 45 \times 3 = 135 \text{ pounds.}$$

$$\text{Company Y cost} = 35 \times 3 + 0.15 \times 250 = 105 + 37.5 = 142.5 \text{ pounds.}$$

Company X is the better deal by £7.50.

Scaling

1. Scale 1:100, wall drawn 3.5 cm.

Solution.

$$3.5 \times 100 = 350 \text{ cm} = 3.5 \text{ m.}$$

2. Map scale 1:50,000, towns 8.4 cm apart.

Solution.

$$8.4 \times 50000 = 420000 \text{ cm} = 4.2 \text{ km.}$$

3. Garden drawn to scale 1:20, drawing measures 4 cm by 3 cm.

Solution.

$$4 \times 20 = 80 \text{ cm,} \quad 3 \times 20 = 60 \text{ cm.}$$

The actual garden is 80 cm by 60 cm, or 0.8 m by 0.6 m.

4. Actual car 4.2 m, model 28 cm.

Solution.

$$4.2 \text{ m} = 420 \text{ cm.}$$

$$28 : 420 = 1 : 15.$$

The scale is 1 : 15.

5. Map scale: 4 cm represents 5 km.

- (a) Cities 15.2 cm apart on the map.

$$15.2 \div 4 = 3.8, \quad 3.8 \times 5 = 19 \text{ km.}$$

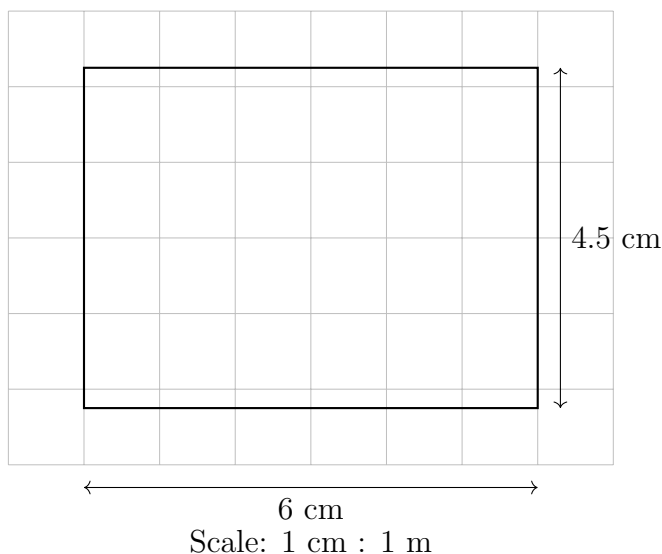
- (b)

$$4 \text{ cm} : 5 \text{ km} = 4 : 500000 = 1 : 125000.$$

- (c) The first map has scale 1 : 125000. The other map also has scale 1 : 125000, so neither map shows more detail; they have the same scale.

6. Floor plan 6 m by 4.5 m, using scale 1 cm : 1 m.

Solution. The drawing should be a rectangle 6 cm by 4.5 cm.



Rates of Change

1. 45 km in 1 hour 30 minutes.

Solution. 1 h 30 min = 1.5 h.

$$45 \div 1.5 = 30 \text{ km/h.}$$

2. 12 litres fills in 3 minutes.

Solution. Rate = $12 \div 3 = 4$ L/min.

$$90 \div 4 = 22.5 \text{ minutes} = 22 \text{ minutes } 30 \text{ seconds.}$$

3. Train: 320 km, 10:15 to 14:45, with a 10-minute stop.

Solution. Total time from 10:15 to 14:45 is

$$4 \text{ h } 30 \text{ min} = 4.5 \text{ h.}$$

Average speed including the stop:

$$320 \div 4.5 = 71.1 \text{ km/h.}$$

Moving time = $4.5 - \frac{10}{60} = 4.5 - \frac{1}{6} = \frac{13}{3}$ hours. Average speed while moving:

$$320 \div \frac{13}{3} = \frac{960}{13} = 73.8 \text{ km/h.}$$

4. Beaker starts with 400 g of chemical A, used at 4 g/min.

Solution. In 4 minutes, used:

$$4 \times 4 = 16 \text{ g.}$$

Remaining:

$$400 - 16 = 384 \text{ g.}$$

5. Sarah drives 30 min at 60 mph, stops 15 min, then drives 45 min at 50 mph.

(a) First distance:

$$60 \times \frac{30}{60} = 30 \text{ miles.}$$

Second distance:

$$50 \times \frac{45}{60} = 37.5 \text{ miles.}$$

Total distance:

$$30 + 37.5 = 67.5 \text{ miles.}$$

(b) Total time including the stop:

$$30 + 15 + 45 = 90 \text{ min} = 1.5 \text{ h.}$$

Average speed:

$$67.5 \div 1.5 = 45 \text{ mph.}$$

Problem Solving With Ratios

1. Red : blue = 3 : 5. After adding 8 red marbles, ratio becomes 5 : 7.

Solution. Let the initial numbers be $3x$ and $5x$.

$$\frac{3x + 8}{5x} = \frac{5}{7}$$

$$7(3x + 8) = 25x$$

$$21x + 56 = 25x$$

$$x = 14.$$

So blue marbles = $5x = 70$.

2. Apples : bananas : oranges = 2 : 5 : 3. After adding 4 apples and 2 bananas, the ratio becomes 4 : 6 : 3.

Solution. Let the initial numbers be $2x, 5x, 3x$. The number of oranges is unchanged and is still 3 parts in the new ratio, so the new scale is also x .

$$2x + 4 = 4x \Rightarrow x = 2,$$

and indeed $5x + 2 = 6x \Rightarrow x = 2$. Thus oranges = $3x = 6$.

3. Sugar : flour = 2 : 9, flour : butter = 3 : 1.

Solution. Make the flour part the same.

$$\text{sugar : flour} = 2 : 9,$$

$$\text{flour : butter} = 3 : 1 = 9 : 3.$$

Therefore

$$\text{sugar : flour : butter} = 2 : 9 : 3.$$

4. Alex : Ben : Chloe = 3 : 4 : 5. Ben receives £40 more than Alex.

Solution. Let the shares be $3x, 4x, 5x$.

$$4x - 3x = x = 40.$$

Total = $3x + 4x + 5x = 12x = 12 \times 40 = 480$. The total sum of money is £480.

5. $AB : BC = 3 : 2$, $AB = 12$ cm.

Solution. $AB = 3$ parts = 12 cm, so one part = 4 cm.

$$BC = 2 \times 4 = 8 \text{ cm.}$$

$$AC = AB + BC = 12 + 8 = 20 \text{ cm.}$$

6. red : yellow = 3 : 4, yellow : blue = 5 : 6.

Solution. Make the yellow part the same.

$$\text{red : yellow} = 3 : 4 = 15 : 20,$$

$$\text{yellow : blue} = 5 : 6 = 20 : 24.$$

So red : yellow : blue = 15 : 20 : 24. Total parts = $15 + 20 + 24 = 59$.

$$\text{fraction blue} = \frac{24}{59}.$$

7. Red : blue = 2 : 3. After adding 36 red and 29 blue marbles, the ratio becomes 4 : 5.

Solution. Let initial numbers be $2x$ and $3x$.

$$\frac{2x + 36}{3x + 29} = \frac{4}{5}$$

$$5(2x + 36) = 4(3x + 29)$$

$$10x + 180 = 12x + 116$$

$$2x = 64$$

$$x = 32.$$

Initial blue marbles = $3x = 96$.