

Worked Solutions: Currency Conversion Graphs

Part 1: Which line shows which currency?

- Q1.** (a) Line A shows Australian dollars.
- (b) £1 buys 5 zloty, the most of the three currencies, so the zloty line is the steepest; that is line B. £1 buys 1.20 euros, the fewest, so the euro line is the least steep; that is line C. £1 buys 2 Australian dollars, which is more than 1.20 euros but less than 5 zloty, so the Australian dollar line is steeper than the euro line and less steep than the zloty line. That is line A.
- (c) Using line A: $y = 2x$. For $x = 7$, $y = 2 \times 7 = 14$. So £7 gives 14 Australian dollars.
- (d) Using line C: $y = 1.2x$. $6 = 1.2x$, so $x = 6 \div 1.2 = 5$. So €6 is £5.
- Q2.** (a) The gradients are the exchange rates: Indian rupees 115, Mexican pesos 25, Norwegian kroner 13.50, Hong Kong dollars 10.50. From steepest to least steep, the lines are: Indian rupees, Mexican pesos, Norwegian kroner, Hong Kong dollars. All pass through (0, 0).
- (b) At £0, you receive 0 of any currency. Since amount = rate $\times$ pounds, when $x = 0$, $y = 0$. So every line passes through the origin (0, 0).
- Q3.** (a) At £20, line P gives $1.3 \times 20 = 26$, line Q gives $1.8 \times 20 = 36$, and line R gives $1.1 \times 20 = 22$. From the table, P matches US dollars (1.30), Q matches Canadian dollars (1.80), and R matches Swiss francs (1.10).
- (b) Singapore dollars: rate 1.70, so $y = 1.7x$. This line lies between P and Q. New Zealand dollars: rate 2.20, so $y = 2.2x$. This is steeper than Q; at £10 it gives 22, while Q gives 18.
- (c) Sam is wrong because line Q is only the steepest of the three lines that were drawn. The table has five currencies, and two have no line. New Zealand dollars have rate 2.20, which is greater than 1.80, so their line would be steeper than Q. Thus Q cannot be New Zealand dollars.

Part 2: Measuring steepness with the gradient

- Q4.** Points on the line: (0, 0) and (5, 120). Change in x : $5 - 0 = 5$. Change in y : $120 - 0 = 120$. Gradient: $120 \div 5 = 24$. Equation: $y = 24x$. The gradient means £1 buys 24 South African rand.
- Q5.** (a) Line S shows Japanese yen. £1 buys more rupees than yen, so the rupee line must be steeper. Line T is steeper, so T is rupees and S is yen.
- (b) Line S: points (0, 0) and (40, 8000). Change in x : 40. Change in y : 8000. Gradient: $8000 \div 40 = 200$. Equation: $y = 200x$. Line T: points (0, 0) and (40, 15000). Change in x : 40. Change in y : 15000. Gradient: $15000 \div 40 = 375$. Equation: $y = 375x$.
- (c) Using S: $y = 200 \times 250 = 50000$. So £250 gives 50 000 yen.
- (d) Using T: $375x = 30000$, so $x = 30000 \div 375 = 80$. So 30 000 rupees is £80.
- Q6.** (a) Since £1 = 5 zloty, each 1 zloty is worth $1 \div 5 = 0.20$ pounds. The table is

zloty	0	10	25	50
£	0	2	5	10

- (b) Draw a straight line from $(0, 0)$ to $(50, 10)$.
- (c) Gradient = $\frac{\text{change in pounds}}{\text{change in zloty}} = \frac{10 - 0}{50 - 0} = \frac{10}{50} = 0.2$.
- (d) The gradient 0.2 means each 1 zloty is worth £0.20, i.e. 20 pence.

Part 3: Flat fees and the y-intercept

- Q7.** Pounds converted: $x - 10$. Dollars received: $y = 1.3(x - 10)$. Expand: $y = 1.3x - 13$. Gradient: 1.3, the exchange rate. y-intercept: -13 , the fee of £10 is worth $1.3 \times 10 = \$13$.
- (a) For £100: $y = 1.3 \times 100 - 13 = 130 - 13 = 117$. So the bank gives \$117.
- (b) Want $y = 130$: $1.3x - 13 = 130$, so $1.3x = 143$, and $x = 143 \div 1.3 = 110$. Jo must exchange £110.
- Q8.** (a) Airport: no fee, rate 1.10, so line M ($y = 1.1x$). High street: £5 fee, rate 1.20, so line L ($y = 1.2(x - 5) = 1.2x - 6$). Online: £15 fee, rate 1.20, so line N ($y = 1.2(x - 15) = 1.2x - 18$). Thus the table is: Airport M, High street L, Online N.
- (b) Lines L and N are parallel because they have the same gradient, 1.2. This is the same exchange rate (€1.20 per £1 left). The different y-intercepts come from the different flat fees.
- (c) L: $y = 1.2x - 6$. M: $y = 1.1x$. N: $y = 1.2x - 18$.
- (d) Lines L and M cross when $1.1x = 1.2x - 6$. Solve: $6 = 0.1x$, so $x = 60$. Then $y = 1.1 \times 60 = 66$. They cross at £60, €66. This means both bureaux give €66 for £60. For less than £60 the airport (M) gives more euros; for more than £60 the high street (L) gives more euros.
- Q9.** (a) The exchange rate is the gradient, 1.8. So £1 buys C\$1.80.
- (b) The y-intercept is -9 , so the fee is worth C\$9. Since each £1 gives C\$1.80, the fee in pounds is $9 \div 1.8 = 5$. So the flat fee is £5.
- (c) If $x = 5$, $y = 1.8 \times 5 - 9 = 9 - 9 = 0$. The whole £5 pays the fee, leaving nothing to convert, so no Canadian dollars are given.
- (d) Second bureau: same rate 1.8, fee £10. The fee is worth $1.8 \times 10 = 18$ Canadian dollars. Equation: $y = 1.8x - 18$ (or $y = 1.8(x - 10)$). Compared with $y = 1.8x - 9$: same gradient, so the lines are parallel; its y-intercept is -18 instead of -9 , so it is 9 Canadian dollars lower at every x .

Part 4: When do two bureaux give the same amount?

- Q10.** (a) For £30: Airport $y = 1.1 \times 30 = 33$. High street $y = 1.2 \times 30 - 6 = 36 - 6 = 30$. The airport gives more (€33 versus €30).
- (b) For £200: Airport $y = 1.1 \times 200 = 220$. High street $y = 1.2 \times 200 - 6 = 240 - 6 = 234$. The high street bureau gives €14 more.
- (c) The high street line has gradient 1.2 and the airport line has gradient 1.1. So for each extra £1, the high street bureau gives $1.2 - 1.1 = 0.10$ euros more than the airport. After the lines cross at £60, the high street bureau is ahead by $0.1(x - 60)$ euros. At £200 this is $0.1 \times (200 - 60) = 0.1 \times 140 = 14$ euros, as found.

Q11. Airport: $y = 4.5x$. Town: $y = 5(x - 3) = 5x - 15$.

x	0	20	40	60
Airport y	0	90	180	270
Town y	-15	85	185	285

The town line is below the airport line at £20 (85 versus 90) and above it at £40 (185 versus 180), so the lines cross between £20 and £40. Solve:

$$4.5x = 5x - 15$$

$$0 = 0.5x - 15$$

$$15 = 0.5x$$

$$30 = x$$

Then $y = 4.5 \times 30 = 135$. So they cross at $(30, 135)$.

- (a) Draw both lines and mark the crossing point $(30, 135)$.
- (b) For £20, the airport gives 90 złoty and the town gives 85 złoty. The airport gives more.
- (c) For £50, the airport gives $4.5 \times 50 = 225$ złoty and the town gives $5 \times 50 - 15 = 250 - 15 = 235$ złoty. The town bureau gives more.

Q12. (a) Airport: $y = 1.15x$. Online: $y = 1.25(x - 8) = 1.25x - 10$.

- (b) Same amount when $1.15x = 1.25x - 10$. Solve: $10 = 0.1x$, so $x = 100$. Then $y = 1.15 \times 100 = 115$. So £100 gives \$115 at both bureaux.
- (c) For £400: Airport $y = 1.15 \times 400 = 460$. Online $y = 1.25 \times 400 - 10 = 500 - 10 = 490$. The online bureau gives \$30 more. So Mia should use the online bureau.

Q13. (a) Bureau E: $y = 190(x - 2) = 190x - 380$. Bureau F: $y = 200(x - 5) = 200x - 1000$.

- (b) Same amount when $190x - 380 = 200x - 1000$. Add 1000: $190x + 620 = 200x$. Subtract $190x$: $620 = 10x$. So $x = 62$. Then $y = 190 \times 62 - 380 = 11780 - 380 = 11400$ yen (or $200 \times 62 - 1000 = 11400$). So £62 gives 11 400 yen at both.
- (c) For £500: Bureau E gives $190 \times 500 - 380 = 95000 - 380 = 94620$ yen. Bureau F gives $200 \times 500 - 1000 = 100000 - 1000 = 99000$ yen. Bureau F gives $99000 - 94620 = 4380$ yen more. Kenji should use Bureau F.

Part 5: Challenges

Q14. (a) G: $y = 1.1(x - 5) = 1.1x - 5.5$, y-intercept -5.5 . H: $y = 1.2(x - 5) = 1.2x - 6$, y-intercept -6 .

- (b) At $x = 5$: G gives $1.1 \times 5 - 5.5 = 5.5 - 5.5 = 0$. H gives $1.2 \times 5 - 6 = 6 - 6 = 0$. So both lines cross the x-axis at $x = 5$, i.e. at $(5, 0)$. This means exchanging exactly £5 pays the flat fee and leaves £0 to convert, so both bureaux give €0.
- (c) Tom is right for any amount over £5. Both lines leave the x-axis at the same point $(5, 0)$. H has gradient 1.2, which is greater than G's 1.1, so for any $x > 5$, H's line is above G's line. Solving $1.1x - 5.5 = 1.2x - 6$ gives $0.5 = 0.1x$, so $x = 5$; the lines only meet at $(5, 0)$. Thus for any actual exchange over £5, H gives more euros. The lower y-intercept of H just reflects that the £5 fee is worth more euros at the better rate, but that part is not usable for real exchanges.

- Q15.** Airport at £300: $y = 1.1 \times 300 = 330$ euros. We need a new bureau with flat fee f and rate $r > 1.1$ such that $r(300 - f) = 330$. For example, choose $r = 1.20$. Then $300 - f = 330 \div 1.2 = 275$, so $f = 300 - 275 = 25$. Deal: rate €1.20 per £1 left, flat fee £25. Equation: $y = 1.2(x - 25) = 1.2x - 30$. At $x = 300$: $y = 1.2 \times 300 - 30 = 360 - 30 = 330$, the same as the airport. Since the new gradient 1.2 is greater than 1.1, for any amount over £300 the new bureau gives more euros. For amounts under £300, the airport gives more euros because the new bureau's line is lower (due to the fee). So anyone exchanging more than £300 should use the new bureau; anyone exchanging less than £300 should use the airport. (Other valid examples: $r = 1.25, f = 36$; $r = 1.32, f = 50$.)
- Q16.** (a) £100 at Bureau K: 2% commission is $0.02 \times 100 = 2$ pounds. So £98 is converted. Euros received: $98 \times 1.25 = 122.5$. So exchanging £100 gives €122.50.
- (b) Aisha is wrong. A commission is a percentage of the amount exchanged, not a fixed amount. If $x = 0$, the commission is £0, so $y = 0$: the line passes through the origin. It does not cross the y-axis below zero. The amount converted is 98% of x , i.e. $0.98x$. Thus $y = 1.25 \times 0.98x = 1.225x$. The gradient is 1.225.
- (c) High street bureau: $y = 1.2(x - 5) = 1.2x - 6$. Bureau K: $y = 1.225x$. Bureau K has a higher y-intercept (0 versus -6) and a steeper gradient ($1.225 > 1.2$), so its line is above the high street line for all positive x . Solving $1.225x = 1.2x - 6$ gives $0.025x = -6$, so $x = -240$, which is not a possible number of pounds to exchange. Thus the lines never cross on a realistic graph. Bureau K is better for every amount, and the answer does not depend on how many pounds are exchanged.