

Currency Conversion Graphs

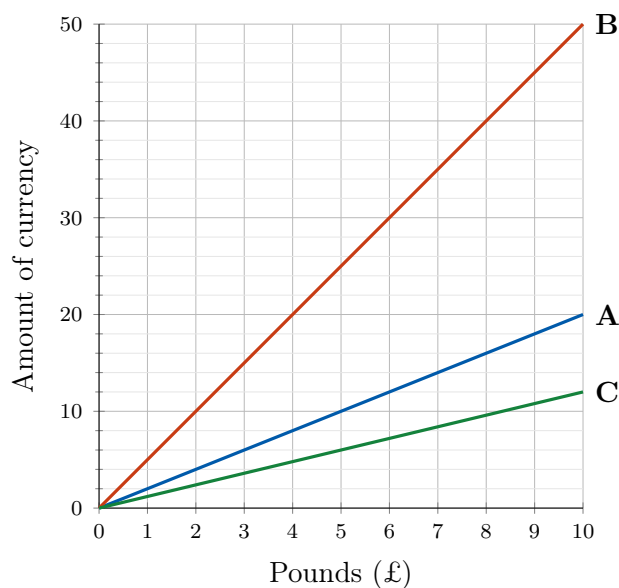
A **bureau de change** is a place where money is changed from one currency into another. The **exchange rate** is the amount of the other currency that £1 buys. When there is no fee, the amount of the other currency is directly proportional to the number of pounds, so the conversion graph is a straight line through the origin. Work through this sheet to find out how the steepness and the position of the line describe the exchange.

The exchange rates in this sheet are rounded. Real exchange rates change every day.

Part 1: Which line shows which currency?

In every graph in this part, pounds are on the x-axis and the amount of another currency is on the y-axis. The graphs do not say which currency each line shows: use the exchange rates in the tables to work it out.

Q1. The graph shows three conversion graphs. Each line shows one of the currencies in the table.



Currency	£1 buys
Euros (€)	1.20
Australian dollars	2
Polish zloty	5

(a) Which line shows Australian dollars?

Line _____

(b) Explain how you decided which line shows Australian dollars. Use each of these words in your answer.

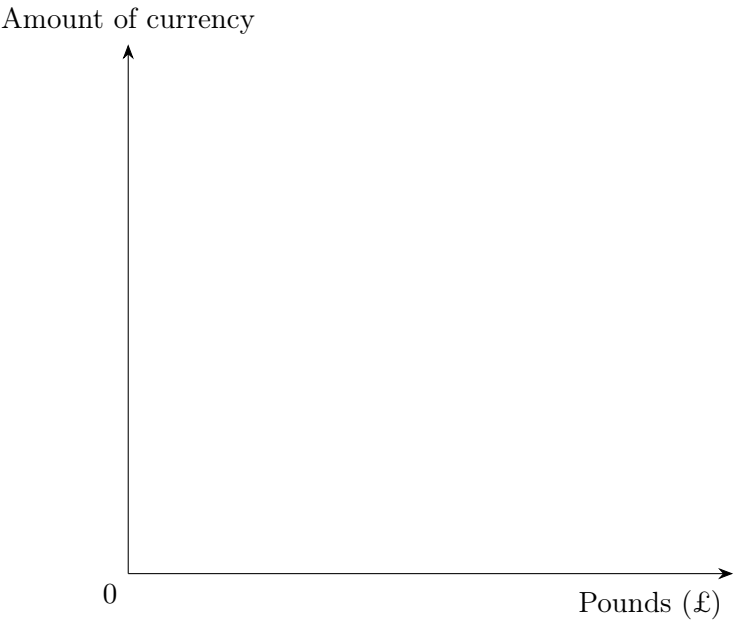
steeper steepest less steep least steep

(c) Use the graph to convert £7 into Australian dollars. _____ Australian dollars

(d) Use the graph to convert €6 into pounds. £_____

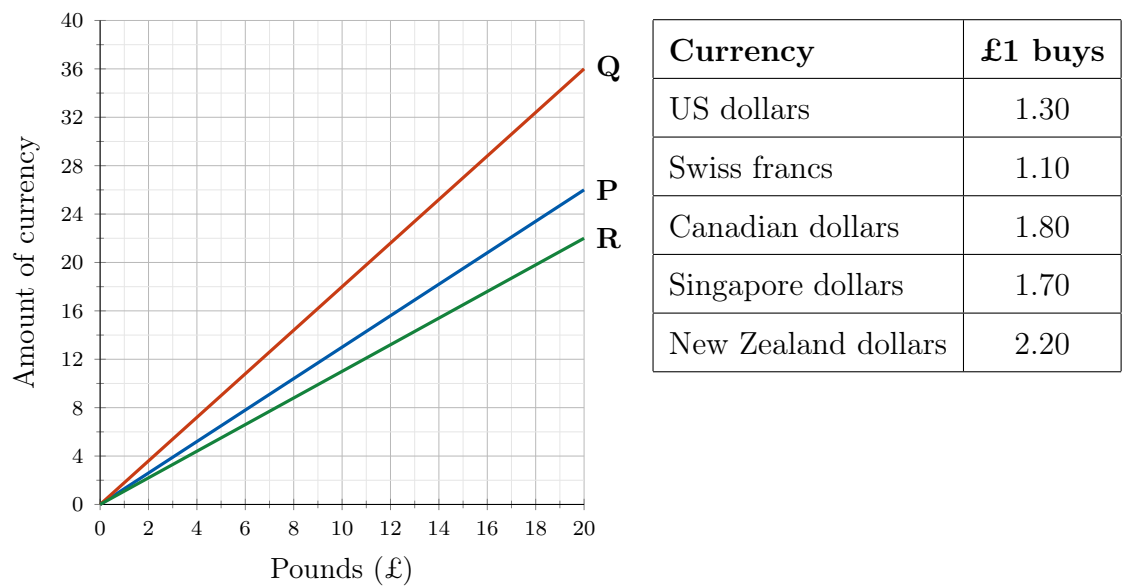
Q2. The table shows how much of four currencies £1 buys. The four conversion graphs are drawn on the same axes, with pounds on the x-axis.

Currency	£1 buys
Norwegian kroner	13.50
Indian rupees	115
Hong Kong dollars	10.50
Mexican pesos	25



- (a) Sketch the four conversion graphs on the axes. The axes do not need a scale, but the lines must be in the right order of steepness. Label each line with its currency.
- (b) Explain why every one of the four lines passes through the origin.

Q3. The graph shows three more conversion graphs, but this time there are five currencies in the table. Two of the currencies do not have a line.



- (a) Write down the currency shown by each line.
Line P: _____ Line Q: _____ Line R: _____
- (b) Draw the lines for Singapore dollars and New Zealand dollars on the graph. Label each line with its currency.
- (c) Sam says: “Line Q is the steepest line, so it must show New Zealand dollars, because £1 buys more New Zealand dollars than any other currency in the table.” Explain why Sam is wrong.

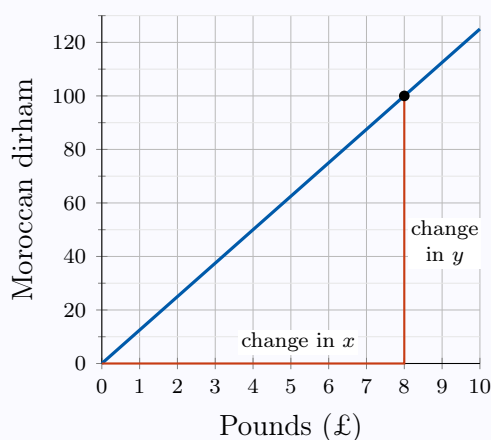
Part 2: Measuring steepness with the gradient

To compare steepness exactly, we measure it with a number called the **gradient**. The gradient of a straight line is how far the line rises for every 1 unit it moves to the right:

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

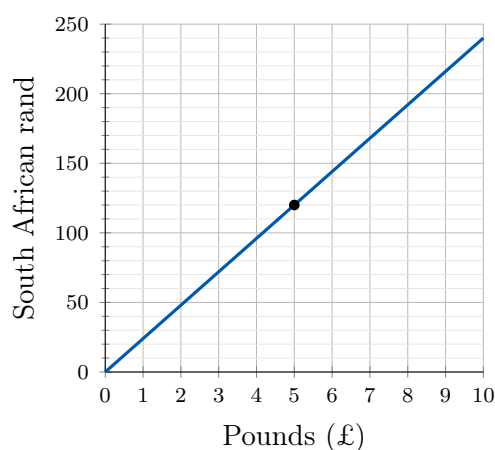
In a conversion graph, x stands for the number of pounds and y stands for the amount of the other currency. Moving 1 unit to the right means adding £1, so the gradient is the exchange rate. A straight line through the origin with gradient m has the equation $y = mx$ (remember $mx = m \times x$).

Worked example: find the gradient of the Moroccan dirham line



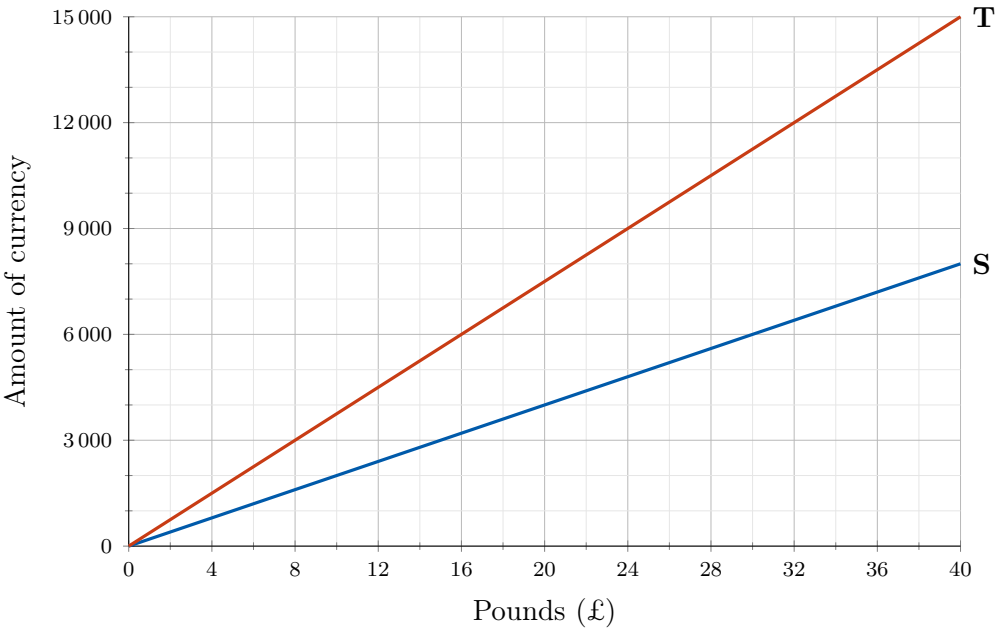
Two points on the line	$(0, 0)$ and $(8, 100)$
Change in x	$8 - 0 = 8$
Change in y	$100 - 0 = 100$
Gradient	$100 \div 8 = 12.5$
Equation of the line	$y = 12.5x$
What the gradient means	£1 buys 12.5 dirham

Q4. The graph shows the conversion graph for South African rand. Fill in the gaps to find its gradient.



Two points on the line	$(0, 0)$ and $(5, \underline{\hspace{2cm}})$
Change in x	$5 - 0 = \underline{\hspace{2cm}}$
Change in y	$\underline{\hspace{2cm}} - 0 = \underline{\hspace{2cm}}$
Gradient	$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
Equation of the line	$y = \underline{\hspace{2cm}} x$
What the gradient means	£1 buys $\underline{\hspace{2cm}}$ rand

Q5. The graph shows two conversion graphs. One line shows Japanese yen and the other shows Pakistani rupees. £1 buys more rupees than yen.



(a) Which line shows Japanese yen? Give a reason for your answer. _____

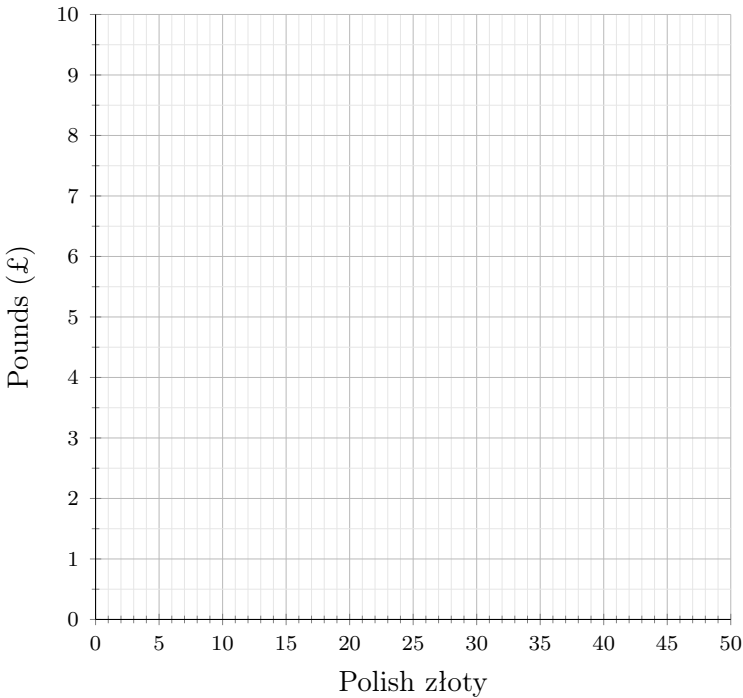
(b) Find the gradient of each line, and write down the equation of each line. Set out your working like the worked example.

Line S		Line T	
Two points on the line	_____	Two points on the line	_____
Change in x	_____	Change in x	_____
Change in y	_____	Change in y	_____
Gradient	_____	Gradient	_____
Equation	_____	Equation	_____

(c) The graph stops at £40. Use the equation of line S to convert £250 into yen.

(d) Use the equation of line T to convert 30 000 rupees into pounds.

Q6. Ola lives in Poland and is visiting London. She draws a conversion graph the other way round, with Polish złoty on the x-axis and pounds on the y-axis. The exchange rate is £1 = 5 złoty.



złoty	0	10	25	50
£	0			

- (a) Complete the table.
- (b) Draw the line on the grid.

(c) Find the gradient of Ola’s line. _____

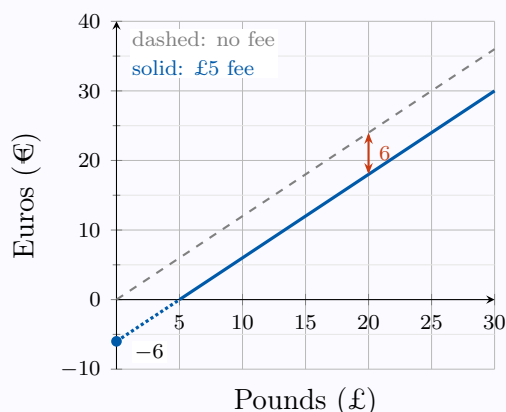
(d) Explain what the gradient of Ola’s line tells her.

Part 3: Flat fees and the y-intercept

Some bureaux de change charge a **flat fee**: a fixed amount that is taken off every exchange, however much money is changed. The fee is taken off first, and the pounds that are left are converted.

A straight line with gradient m that crosses the y-axis at c has the equation $y = mx + c$. The number c is called the **y-intercept**. For a bureau with a flat fee, the gradient is still the exchange rate, and the y-intercept is negative.

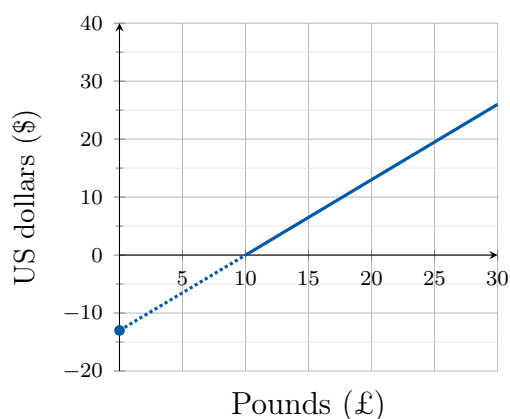
Worked example: £5 fee, and €1.20 for every £1 left



Pounds converted $x - 5$
Euros received $y = 1.2(x - 5)$
Expand the brackets $y = 1.2x - 6$
Gradient 1.2, the exchange rate
y-intercept -6 , the fee of £5
 is worth $1.2 \times 5 = €6$

The fee line is parallel to the no-fee line, because the exchange rate is the same. It is €6 lower at every point, because the fee is worth €6. The dotted part of the line is only there to show where the line meets the y-axis: exchanging less than £5 gives nothing at all.

Q7. A bank charges a flat fee of £10, and gives \$1.30 for every £1 left. Fill in the gaps to find the equation of its conversion graph.

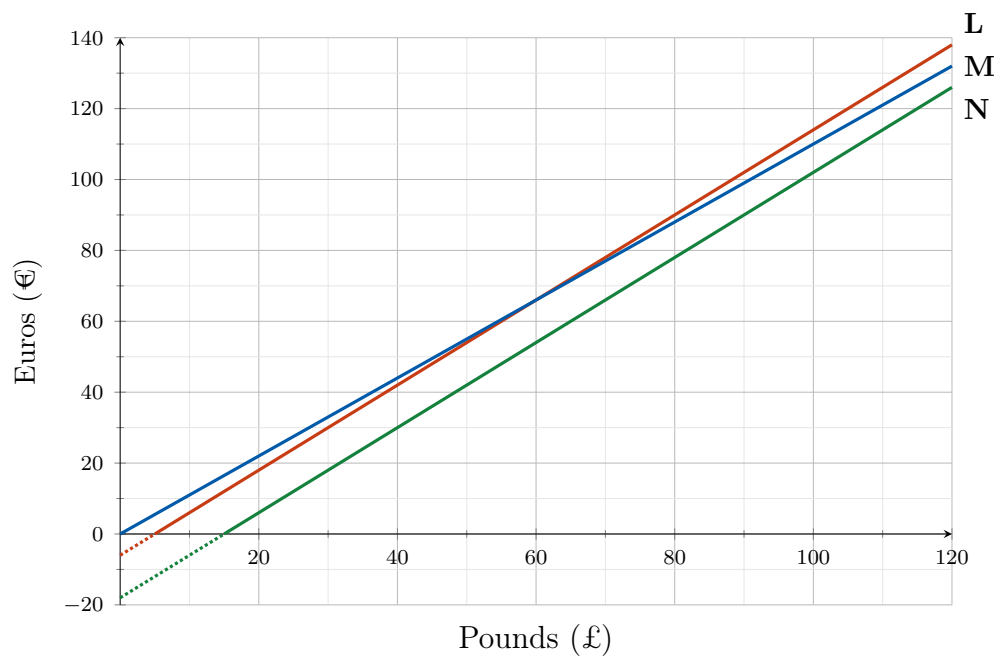


Pounds converted $x - \underline{\hspace{2cm}}$
Dollars received $y = \underline{\hspace{2cm}}(x - \underline{\hspace{2cm}})$
Expand the brackets $y = \underline{\hspace{2cm}}x - \underline{\hspace{2cm}}$
Gradient $\underline{\hspace{2cm}}$, the exchange rate
y-intercept $\underline{\hspace{2cm}}$, the fee of £ $\underline{\hspace{2cm}}$
 is worth $\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \$\underline{\hspace{2cm}}$

(a) How many dollars does the bank give for £100? \$

(b) Jo wants to receive exactly \$130. How many pounds must she exchange? £

Q8. The graph shows the conversion graphs for three bureaux de change. As in the worked example, the dotted part of each line only shows where the line meets the y-axis.



Bureau	Flat fee	Euros for every £1 left	Line
Airport	none	1.10	
High street	£5	1.20	
Online	£15	1.20	

- (a) Complete the table to match each bureau to its line.
- (b) Two of the lines are parallel. Explain why.
- (c) Write down the equation of each line in the form $y = mx + c$.
Line L: _____ Line M: _____ Line N: _____
- (d) Lines L and M cross. Use the graph to find the number of pounds where they cross, and explain what the crossing point tells you about the airport and the high street bureaux.

Q9. A bureau's conversion graph for Canadian dollars has the equation

$$y = 1.8x - 9$$

where x is the number of pounds exchanged and y is the number of Canadian dollars received.

- (a) What is the exchange rate at this bureau? _____
- (b) What flat fee does the bureau charge, in pounds? Show how you worked it out.
- (c) Explain why this bureau gives no Canadian dollars at all to someone who exchanges £5.
- (d) A second bureau has the same exchange rate, but charges a flat fee of £10. Write down the equation of its conversion graph, and describe how its line compares with the line $y = 1.8x - 9$.

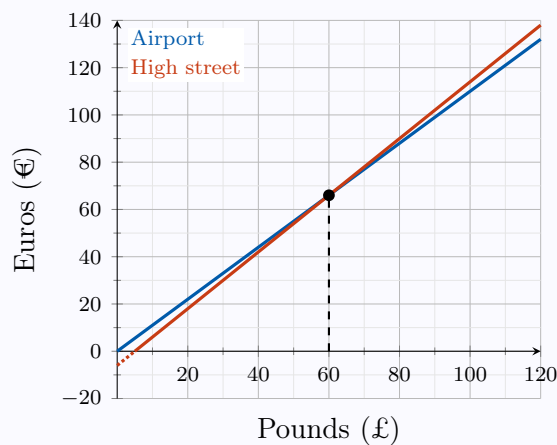
Part 4: When do two bureaux give the same amount?

Where two conversion graphs cross, the two bureaux give the same amount of currency for the same number of pounds. To find the crossing point exactly, we set the two expressions for y equal to each other and solve the equation. Finding the values of x and y that make two equations true at the same time is called solving **simultaneous equations**.

Worked example: the airport and the high street bureau from Q8

Airport: no fee, €1.10 for every £1, so $y = 1.1x$.
High street: £5 fee, €1.20 for every £1 left, so $y = 1.2(x - 5) = 1.2x - 6$.

Pounds, x	0	40	80	120
Airport, y	0	44	88	132
High street, y	-6	42	90	138



The high street line is below the airport line at £40 and above it at £80, so the lines cross between £40 and £80.

Same amount when

$1.1x = 1.2x - 6$

Subtract $1.1x$ from both sides

$0 = 0.1x - 6$

Add 6 to both sides

$6 = 0.1x$

Multiply both sides by 10

$60 = x$

Euros received

$y = 1.1 \times 60 = 66$

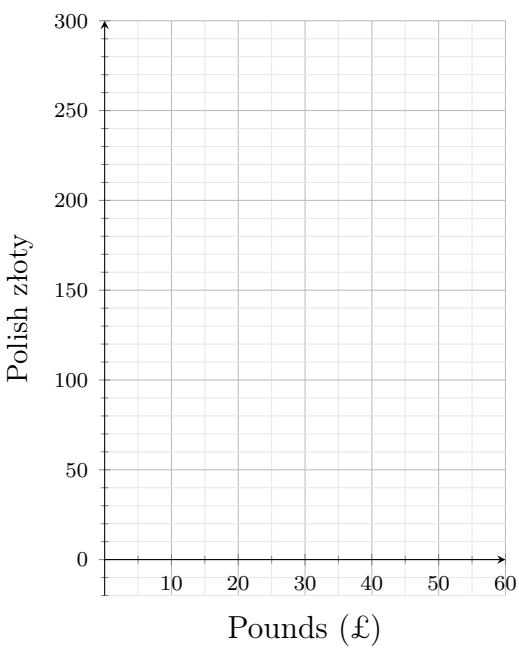
Both bureaux give €66 for £60. For less than £60 the airport gives more euros, because its line is higher. For more than £60 the high street bureau gives more euros, because its line is steeper and is now the higher line.

- Q10.** Use the worked example to answer these.
- (a) How many euros does each bureau give for £30? Which bureau gives more?
 - (b) How many euros does each bureau give for £200? Which bureau gives more, and how many more?
 - (c) Once the lines have crossed, the difference between the two bureaux gets bigger as the number of pounds goes up. Use the gradients of the two lines to explain why.

Q11. An airport bureau and a bureau in town both change pounds into Polish złoty. Fill in the gaps in the equations, the table and the working, as in the worked example.

Airport: no fee, 4.50 złoty for every £1, so $y = \rule{1cm}{0.4pt} x$.
Town: £3 fee, 5 złoty for every £1 left, so $y = 5(x - \rule{1cm}{0.4pt}) = \rule{1cm}{0.4pt} x - \rule{1cm}{0.4pt}$.

Pounds, x	0	20	40	60
Airport, y	0			
Town, y				



The town line is $\rule{1cm}{0.4pt}$ the airport line at £20 and $\rule{1cm}{0.4pt}$ it at £40, so the lines cross between £20 and £40.

Same amount when $\rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$

Subtract $4.5x$ from both sides $0 = \rule{1cm}{0.4pt} x - \rule{1cm}{0.4pt}$

Add $\rule{1cm}{0.4pt}$ to both sides $\rule{1cm}{0.4pt} = 0.5x$

Multiply both sides by $\rule{1cm}{0.4pt}$ $\rule{1cm}{0.4pt} = x$

Złoty received $y = \rule{1cm}{0.4pt}$

- (a) Draw both lines on the grid, and mark the point where they cross.
- (b) Which bureau gives more złoty for £20? $\rule{2cm}{0.4pt}$
- (c) Which bureau gives more złoty for £50? $\rule{2cm}{0.4pt}$

- Q12.** The airport bureau gives \$1.15 for every £1 and charges no fee. An online bureau charges a flat fee of £8 and gives \$1.25 for every £1 left.
- (a) Write down the equation of each conversion graph.
- (b) Find the number of pounds for which both bureaux give the same number of dollars, and how many dollars that is.
- (c) Mia is exchanging £400 for a holiday in New York. Which bureau should she use, and how many more dollars will she get?
- Q13.** Two bureaux both charge a flat fee. Bureau E charges £2 and gives 190 Japanese yen for every £1 left. Bureau F charges £5 and gives 200 yen for every £1 left.
- (a) Show that the equation for Bureau E is $y = 190x - 380$, and write down the equation for Bureau F.
- (b) Find the number of pounds for which both bureaux give the same number of yen.
- (c) Kenji has £500 to exchange. Which bureau should he use, and how many more yen will he get?

Part 5: Challenges

Q14. Two bureaux both charge a flat fee of £5. Bureau G gives €1.10 for every £1 left, and Bureau H gives €1.20 for every £1 left.

(a) Write down the equation of each conversion graph, and the y-intercept of each line.

(b) Show that both lines cross the x-axis at $x = 5$, and explain what this point means.

(c) Tom says: “If two bureaux charge the same flat fee, the bureau with the better exchange rate always gives more euros.” Use your answers to parts (a) and (b) to decide whether Tom is right.

Q15. The airport bureau gives €1.10 for every £1 and charges no fee. Design a deal for a new bureau, with a flat fee and a better exchange rate, so that the new bureau and the airport give the same number of euros when exactly £300 is exchanged. Show that your deal works, and explain who should use the new bureau rather than the airport.

Q16. Some bureaux charge a **commission** instead of a flat fee. A commission is a percentage of the pounds exchanged. Bureau K takes 2% commission, and gives €1.25 for every £1 left.

- (a) Show that exchanging £100 at Bureau K gives €122.50.

- (b) Aisha says: “A commission is a kind of fee, so the line for Bureau K crosses the y-axis below zero, like the line for the high street bureau.” Explain whether Aisha is right, and find the gradient of the line for Bureau K.

- (c) Is Bureau K better or worse than the high street bureau (£5 fee, €1.20 for every £1 left)? Does your answer depend on how many pounds are exchanged? Explain using the equations of the two lines.

Answers

Q1. (a) Line A. (b) For example: £1 buys more złoty than either of the other currencies, so each £1 adds the most to the złoty line, and the złoty line is the steepest line, B. £1 buys the fewest euros, so the euro line is the least steep line, C. £1 buys 2 Australian dollars, which is more than €1.20 and fewer than 5 złoty, so the Australian dollar line is steeper than the euro line and less steep than the złoty line. That is line A. (c) 14 Australian dollars. (d) £5, reading from line C.

Q2. (a) From the steepest line to the least steep: Indian rupees (115), Mexican pesos (25), Norwegian kroner (13.50), Hong Kong dollars (10.50). (b) £0 buys none of any currency, so every line passes through (0,0). The amount of each currency is directly proportional to the number of pounds.

Q3. (a) At £20, line P is at 26, line Q is at 36 and line R is at 22. $26 \div 20 = 1.3$, $36 \div 20 = 1.8$ and $22 \div 20 = 1.1$, so P shows US dollars, Q shows Canadian dollars and R shows Swiss francs. (b) Singapore dollars: a straight line from (0,0) through (10,17) and (20,34), between lines P and Q. New Zealand dollars: a straight line from (0,0) through (10,22), which reaches the top of the graph at about £18. It is steeper than line Q. (c) Line Q is only the steepest of the three lines that were drawn. Two currencies in the table had no line, and the New Zealand dollar line from part (b) is steeper than line Q: at £10 it is at 22, while line Q is at 18.

Q4. (0,0) and (5,120). Change in x : 5. Change in y : $120 - 0 = 120$. Gradient: $120 \div 5 = 24$. Equation: $y = 24x$. £1 buys 24 rand.

Q5. (a) Line S. £1 buys fewer yen than rupees, so the yen line is the less steep line. (b) Line S: for example (0,0) and (40,8000); change in x is 40, change in y is 8000, gradient $8000 \div 40 = 200$, so $y = 200x$. Line T: for example (0,0) and (40,15000); change in x is 40, change in y is 15000, gradient $15000 \div 40 = 375$, so $y = 375x$. (c) $y = 200 \times 250 = 50000$ yen. (d) $375x = 30000$, so $x = 30000 \div 375 = 80$. That is £80.

Q6. (a) £0, £2, £5, £10. (b) A straight line from (0,0) to (50,10). (c) $10 \div 50 = 0.2$. (d) 1 złoty

is worth £0.20, which is 20p.

Q7. Pounds converted: $x - 10$. Dollars received: $y = 1.3(x - 10)$. Expanded: $y = 1.3x - 13$. Gradient: 1.3. y -intercept: -13 , the fee of £10 is worth $1.3 \times 10 = \$13$. (a) $1.3 \times 100 - 13 = \$117$. (b) $1.3x - 13 = 130$, so $1.3x = 143$ and $x = 110$. Jo must exchange £110.

Q8. (a) Airport: M. High street: L. Online: N. (b) Lines L and N both give €1.20 for every £1 left, so both have gradient 1.2. (c) L: $y = 1.2x - 6$. M: $y = 1.1x$. N: $y = 1.2x - 18$. (d) £60. The airport and the high street bureau give the same number of euros (€66) for £60. For less than £60 the airport gives more, and for more than £60 the high street bureau gives more.

Q9. (a) C\$1.80 for every £1. (b) The fee is worth C\$9, so the fee is $9 \div 1.8 = £5$. (c) The whole £5 is taken as the fee, so there are no pounds left to convert: $1.8 \times 5 - 9 = 0$. (d) $y = 1.8x - 18$. The line is parallel to $y = 1.8x - 9$, because the gradient is the same, and it is 9 lower at every point, because its y -intercept is -18 instead of -9 .

Q10. (a) The airport gives $1.1 \times 30 = €33$ and the high street bureau gives $1.2 \times 30 - 6 = €30$, so the airport gives more. (b) The airport gives $1.1 \times 200 = €220$ and the high street bureau gives $1.2 \times 200 - 6 = €234$, so the high street bureau gives €14 more. (c) The high street line has gradient 1.2 and the airport line has gradient 1.1, so for every extra £1 the high street bureau gives €0.10 more than the airport. After the lines cross at £60, the difference grows by €0.10 for every £1: at £200 it is $0.1 \times (200 - 60) = €14$.

Q11. Airport: $y = 4.5x$. Town: $y = 5(x - 3) = 5x - 15$. Table: airport 0, 90, 180, 270; town -15 , 85, 185, 285. The town line is below the airport line at £20 and above it at £40. Working: $4.5x = 5x - 15$; $0 = 0.5x - 15$; add 15: $15 = 0.5x$; multiply by 2: $30 = x$; $y = 4.5 \times 30 = 135$. (a) The lines cross at (30,135). (b) The airport: 90 złoty against 85 złoty. (c) The town bureau: 235 złoty against 225 złoty.

Q12. (a) Airport: $y = 1.15x$. Online: $y = 1.25(x -$

8) = $1.25x - 10$. (b) $1.15x = 1.25x - 10$, so $10 = 0.1x$ and $x = 100$. Both bureaux give \$115 for £100. (c) The online bureau: $1.25 \times 400 - 10 = \$490$, against $1.15 \times 400 = \$460$ at the airport. She gets \$30 more.

Q13. (a) E: $y = 190(x - 2) = 190x - 380$. F: $y = 200(x - 5) = 200x - 1000$. (b) $190x - 380 = 200x - 1000$, so $620 = 10x$ and $x = 62$. Both bureaux give 11 400 yen for £62. (c) Bureau F: $200 \times 500 - 1000 = 99\,000$ yen, against $190 \times 500 - 380 = 94\,620$ yen at Bureau E. He gets 4380 more yen.

Q14. (a) G: $y = 1.1(x - 5) = 1.1x - 5.5$, y-intercept -5.5 . H: $y = 1.2(x - 5) = 1.2x - 6$, y-intercept -6 . (b) $1.1 \times 5 - 5.5 = 0$ and $1.2 \times 5 - 6 = 0$, so both lines pass through $(5, 0)$. Exchanging exactly £5 pays the fee and leaves nothing to convert, so both bureaux give no euros. (c) Tom is right for every amount over £5. Both lines leave the x-axis at the same point, $(5, 0)$, and H's line is steeper, so H's line is above G's line for every amount over £5. Solving $1.1x - 5.5 = 1.2x - 6$ gives $x = 5$, so the lines only meet at $(5, 0)$. H's y-intercept is lower, but that part of the line is dotted and does not describe a real exchange.

Q15. Many answers. The airport gives $1.1 \times 300 = \text{€}330$ for £300, so the new deal needs rate $\times (300 - \text{fee}) = 330$. For example: €1.20 with a £25 fee, since $1.2 \times 275 = 330$; €1.25 with a £36 fee, since $1.25 \times 264 = 330$; €1.32 with a £50 fee, since $1.32 \times 250 = 330$. The new bureau's line is steeper, so it gives more euros for any amount over £300. Anyone exchanging more than £300 should use the new bureau, and anyone exchanging less should use the airport.

Q16. (a) 2% of £100 is £2, so £98 is converted: $98 \times 1.25 = 122.5$. (b) Aisha is wrong. The commission is 2% of the pounds exchanged, so exchanging £0 costs nothing and gives €0: the line passes through the origin. 98% of the pounds are converted, so $y = 1.25 \times 0.98x = 1.225x$, and the gradient is 1.225. (c) Bureau K is better for every amount. K: $y = 1.225x$. High street: $y = 1.2x - 6$. K's line starts higher (0 is above -6) and is steeper (1.225 is more than 1.2), so it is above the high street line for every number of pounds. Solving $1.225x = 1.2x - 6$ gives $x = -240$, which is not an amount that can be exchanged, so the lines never cross on the graph.