

Worked Solutions: Ratio Extension Problems

Simplifying Ratios Past

1. In a survey, 60% of people preferred tea and 40% preferred coffee. Write the ratio of tea drinkers to coffee drinkers in its simplest form.

Solution:

$$\text{Tea : Coffee} = 60\% : 40\% = 60 : 40 = \frac{60}{20} : \frac{40}{20} = 3 : 2$$

2. A shirt is on sale for 30% off. The discount amount is £12. What is the ratio of the discount amount to the original price? Simplify your answer.

Solution:

Let the original price be P . Discount = 30% of P = £12.

$$0.3P = 12 \quad \Rightarrow \quad P = \frac{12}{0.3} = 40$$

Original price = £40. Discount = £12.

$$\text{Ratio} = 12 : 40 = \frac{12}{4} : \frac{40}{4} = 3 : 10$$

3. A battery has lost 15% of its charge. What is the ratio of the charge lost to the charge remaining? Simplify your answer.

Solution:

Let total charge = 100%. Charge lost = 15%. Charge remaining = 85%.

$$\text{Lost : Remaining} = 15 : 85 = \frac{15}{5} : \frac{85}{5} = 3 : 17$$

4. In a year, a company's profits were allocated as follows: 45% was reinvested, 20% was paid as dividends, and the rest was used for employee bonuses.

- (a) What percentage was used for employee bonuses?

Solution:

$$100\% - (45\% + 20\%) = 100\% - 65\% = 35\%$$

- (b) Express the ratio Reinvestment : Dividends : Bonuses in its simplest form.

Solution:

$$45 : 20 : 35 = \frac{45}{5} : \frac{20}{5} : \frac{35}{5} = 9 : 4 : 7$$

5. The population of a town is 12,500. After a census, it's found that the population has increased by 18%. What is the ratio of the new population to the original population? Express your answer in the form $a : b$, where a and b are integers.

Solution:

Increase = 18% of 12,500 = $0.18 \times 12500 = 2250$.

New population = $12500 + 2250 = 14750$.

$$\text{Ratio} = 14750 : 12500 = \frac{14750}{250} : \frac{12500}{250} = 59 : 50$$

Simplifying Ratios Future

6. The two rectangles below are similar. Find the ratio of the perimeter of rectangle A to the perimeter of rectangle B. Then, find the ratio of the area of rectangle A to the area of rectangle B.

Solution:

Rectangle A: $2 \text{ cm} \times 1 \text{ cm}$. Rectangle B: $4 \text{ cm} \times 2 \text{ cm}$.

Scale factor from A to B: $\frac{4}{2} = 2$ (or $\frac{2}{1} = 2$).

Perimeter ratio = scale factor = $2 : 1$.

Area ratio = (scale factor)² = $2^2 : 1^2 = 4 : 1$.

7. A square has a side length of s . Another square has a side length of $3s$. What is the ratio of the perimeter of the smaller square to the larger square? What is the ratio of their areas?

Solution:

Perimeter smaller = $4s$, perimeter larger = $4(3s) = 12s$.

Perimeter ratio = $4s : 12s = 1 : 3$.

Area smaller = s^2 , area larger = $(3s)^2 = 9s^2$.

Area ratio = $s^2 : 9s^2 = 1 : 9$.

8. A right-angled triangle has a base of 8 cm and a height of 6 cm. A similar triangle has a base of 12 cm.

- (a) What is the scale factor from the smaller to the larger triangle?

Solution:

$$\text{Scale factor} = \frac{12}{8} = 1.5 = \frac{3}{2}$$

- (b) What is the ratio of the perimeter of the smaller triangle to the larger triangle?

Solution:

Perimeter ratio = $1 : \frac{3}{2} = 2 : 3$.

- (c) What is the ratio of the area of the smaller triangle to the larger triangle?

Solution:

Area ratio = $1^2 : \left(\frac{3}{2}\right)^2 = 1 : \frac{9}{4} = 4 : 9$.

9. The diagram below shows a large rectangle with a smaller rectangle cut out of it. Find the ratio of the area of the shaded region to the area of the unshaded (cut-out) rectangle.

Solution:

Large rectangle area = $8 \times 6 = 48 \text{ m}^2$.

Unshaded rectangle area = $4 \times 2 = 8 \text{ m}^2$.

Shaded area = $48 - 8 = 40 \text{ m}^2$.

$$\text{Ratio} = 40 : 8 = 5 : 1$$

10. A circular pond has a radius of r . A path is built around it, creating a larger circle with a radius of R , where $R = 2r$.

- (a) Find the ratio of the circumference of the pond to the circumference of the outer edge of the path.

Solution:

Circumference pond = $2\pi r$, circumference outer edge = $2\pi R = 2\pi(2r) = 4\pi r$.

Ratio = $2\pi r : 4\pi r = 1 : 2$.

- (b) Find the ratio of the area of the pond to the total area enclosed by the outer edge of the path.

Solution:

Area pond = πr^2 , total area = $\pi R^2 = \pi(2r)^2 = 4\pi r^2$.

Ratio = $\pi r^2 : 4\pi r^2 = 1 : 4$.

Sharing Into Ratios Past

11. A sum of £80 is shared between two people. Person A receives 60% of the total. What is the ratio of their shares, and how much does each person receive?

Solution:

Person A: 60%, Person B: 40%. Ratio = $60 : 40 = 3 : 2$.

Amount A = $0.6 \times 80 = 48$.

Amount B = $80 - 48 = 32$.

12. In a class, 40% of students are boys and the rest are girls. The school fund of £150 is to be shared between boys and girls in the same ratio as their numbers in the class. How much do the boys get?

Solution:

Boys: 40%, Girls: 60%. Ratio = $40 : 60 = 2 : 3$.

Total parts = $2 + 3 = 5$.

Boys' share = $\frac{2}{5} \times 150 = 60$.

13. A bonus of £2,500 is shared between three departments in proportion to their percentage contribution to company profits. Department A contributed 25%, Department B 35%, and Department C the remainder. How much does each department receive, and what is the ratio of their shares in simplest form?

Solution:

Department C = $100\% - (25\% + 35\%) = 40\%$.

Ratio = $25 : 35 : 40 = 5 : 7 : 8$ (dividing by 5).

Total parts = $5 + 7 + 8 = 20$.

Amount A = $\frac{5}{20} \times 2500 = 625$.

$$\text{Amount B} = \frac{7}{20} \times 2500 = 875.$$

$$\text{Amount C} = \frac{\frac{8}{20}}{\frac{8}{20}} \times 2500 = 1000.$$

14. In a recipe, the dry ingredients (flour and sugar) are mixed in the ratio 3:2 by weight. If the flour makes up 30% of the total mixture (including 400g of wet ingredients), find the weight of flour and sugar used.

Solution:

Let total mixture weight = T . Flour = 30% of T .

Dry ingredients = flour + sugar. Flour : sugar = 3:2, so flour = $\frac{3}{5}$ of dry weight.

But flour = $0.3T$, so dry weight = $\frac{5}{3} \times 0.3T = 0.5T$.

Wet ingredients = $T - 0.5T = 0.5T = 400$ g.

Thus $T = 800$ g.

Flour = $0.3 \times 800 = 240$ g.

Dry weight = $0.5 \times 800 = 400$ g.

Sugar = $400 - 240 = 160$ g.

15. An inheritance is shared between three children. The eldest receives 50% of the total, the middle child receives 40% of what the eldest gets, and the youngest receives the remaining £24,000. Find the total inheritance and the ratio of their shares in the form *Eldest : Middle : Youngest*.

Solution:

Let total = T . Eldest = $0.5T$. Middle = $0.4 \times (0.5T) = 0.2T$.

Youngest = $T - (0.5T + 0.2T) = 0.3T = 24000$.

So $T = \frac{24000}{0.3} = 80,000$.

Eldest = $0.5 \times 80000 = £40,000$. Middle = $0.2 \times 80000 = £16,000$. Youngest = £24,000.

Ratio = $40000 : 16000 : 24000 = 40 : 16 : 24 = 5 : 2 : 3$ (dividing by 8).

Sharing Into Ratios Future

16. The perimeter of a rectangle is 30 cm. The sides are in the ratio 2:3. Find the area of the rectangle.

Solution:

Let sides be $2x$ and $3x$. Perimeter = $2(2x + 3x) = 10x = 30 \implies x = 3$.

Sides: 6 cm and 9 cm. Area = $6 \times 9 = 54$ cm².

17. The area of a rectangle is 54 cm². The sides are in the ratio 2:3. Find the perimeter of the rectangle.

Solution:

Let sides be $2x$ and $3x$. Area = $2x \times 3x = 6x^2 = 54 \implies x^2 = 9 \implies x = 3$.

Sides: 6 cm and 9 cm. Perimeter = $2(6 + 9) = 30$ cm.

18. A right-angled triangle has sides in the ratio 3:4:5. The area of the triangle is 96 cm². Find the perimeter of the triangle.

Solution:

Let sides be $3x, 4x, 5x$. The two shorter sides are perpendicular.

Area = $\frac{1}{2} \times 3x \times 4x = 6x^2 = 96 \implies x^2 = 16 \implies x = 4$.

Sides: 12 cm, 16 cm, 20 cm. Perimeter = $12 + 16 + 20 = 48$ cm.

19. A garden is in the shape of an L-shape made from two rectangles. The first rectangle is 3m by 2m. The second rectangle has sides in the ratio 3:4, If the total area of the garden is 54 m^2 , find the dimensions of the second rectangle.

Solution:

Area of first rectangle $= 3 \times 2 = 6 \text{ m}^2$.

Area of second rectangle $= 54 - 6 = 48 \text{ m}^2$.

Let sides of second rectangle be $3y$ and $4y$.

Area $= 3y \times 4y = 12y^2 = 48 \implies y^2 = 4 \implies y = 2$.

Dimensions: $3 \times 2 = 6 \text{ m}$ and $4 \times 2 = 8 \text{ m}$.

20. A circular pond is surrounded by a path. The radius of the pond and the width of the path are in the ratio 2:1. If the total area (pond + path) is $99\pi \text{ m}^2$, find the width of the path.

Solution:

Let pond radius $= 2x$, path width $= x$. Then outer radius $= 2x + x = 3x$.

Total area $= \pi(3x)^2 = 9\pi x^2 = 99\pi \implies x^2 = 11 \implies x = \sqrt{11} \approx 3.32 \text{ m}$.

Width of path $= \sqrt{11} \text{ m}$.

Ratio Problem Solving Past

21. In a class, the ratio of boys to girls is 3 : 2. 40% of the boys wear glasses. 20% of the girls wear glasses, what percentage of the *whole class* wear glasses?

Solution:

Let number of boys $= 3k$, girls $= 2k$, total $= 5k$.

Boys with glasses $= 0.4 \times 3k = 1.2k$. Girls with glasses $= 0.2 \times 2k = 0.4k$.

Total with glasses $= 1.2k + 0.4k = 1.6k$.

Percentage $= \frac{1.6k}{5k} \times 100\% = 32\%$.

22. The price of a rare coin is increased by 20%. The new ratio of its price to its original price is $a : b$. Write the ratio $a : b$ in its simplest form.

Solution:

Let original price $= P$. New price $= 1.2P$.

Ratio new : original $= 1.2P : P = 1.2 : 1 = 12 : 10 = 6 : 5$.

23. A shop sells only two types of fruit: apples and oranges. The ratio of apples to oranges is 5 : 3. 30% of the apples are green, and the rest are red. 60% of the oranges are from Spain, and the rest are from Morocco. What is the ratio of **green apples** to **oranges from Morocco**?

Solution:

Let apples $= 5k$, oranges $= 3k$.

Green apples $= 0.3 \times 5k = 1.5k$.

Oranges from Morocco $= (1 - 0.6) \times 3k = 0.4 \times 3k = 1.2k$.

Ratio $= 1.5k : 1.2k = 15 : 12 = 5 : 4$.

24. In a library, the ratio of fiction to non-fiction books is 7 : 3. After a donation, the number of fiction books increased by 15% and the number of non-fiction books

decreased by 10%. Find the new ratio of fiction to non-fiction books. Give your answer in the form $A : B$ where A and B are integers.

Solution:

Let fiction = $7k$, non-fiction = $3k$.

New fiction = $7k \times 1.15 = 8.05k$.