

Worked Solutions: Applied Ratio Extension Problems

Recipe Questions Past

1. Increasing muffin yield by 50%:

Original yield: 12 muffins.

Increase by 50%: $12 \times 1.5 = 18$ muffins.

Scaling factor: $\frac{18}{12} = 1.5$.

New quantities:

- Flour: $300 \times 1.5 = 450$ g
- Sugar: $200 \times 1.5 = 300$ g
- Eggs: $2 \times 1.5 = 3$ eggs (or 150 g)
- Milk: $250 \times 1.5 = 375$ ml

2. Reducing sauce recipe to 75%:

Scaling factor: 0.75.

New quantities:

- Tomato passata: $400 \times 0.75 = 300$ ml
- Garlic: $2 \times 0.75 = 1.5$ cloves (or 15 g)
- Olive oil: $100 \times 0.75 = 75$ ml
- Basil: $50 \times 0.75 = 37.5$ g

3. Serving 10 people from a 6-person cake recipe:

Scaling factor: $\frac{10}{6} = \frac{5}{3} \approx 1.6667$.

Percentage increase: $(\frac{5}{3} - 1) \times 100\% = \frac{2}{3} \times 100\% \approx 66.67\%$.

New quantities:

- Flour: $240 \times \frac{5}{3} = 400$ g
- Butter: $180 \times \frac{5}{3} = 300$ g
- Sugar: $150 \times \frac{5}{3} = 250$ g
- Eggs: $3 \times \frac{5}{3} = 5$ eggs (or 250 g)

4. Finding original pasta mass:

Let original mass = x g.

Increased by 40%: $x \times 1.4 = 560$

$$x = \frac{560}{1.4} = 400 \text{ g.}$$

Original mass: 400 g.

5. Bread recipe reduction:

(a) Original flour: 1000 g, new flour: 850 g.

Reduction factor: $\frac{850}{1000} = 0.85$.

Percentage reduction: $(1 - 0.85) \times 100\% = 15\%$.

(b) New quantities:

- Water: $600 \times 0.85 = 510$ ml
- Salt: $20 \times 0.85 = 17$ g
- Yeast: $15 \times 0.85 = 12.75$ g

Recipe Questions Future

6. Square brownie tray enlargement:

Original area: $20 \times 20 = 400 \text{ cm}^2$.

New area: $25 \times 25 = 625 \text{ cm}^2$.

Scaling factor: $\frac{625}{400} = 1.5625$.

New quantities:

- Chocolate: $200 \times 1.5625 = 312.5$ g
- Butter: $150 \times 1.5625 = 234.375$ g
- Eggs: $3 \times 1.5625 = 4.6875 \approx 4.7$ eggs (practically 5 eggs)

7. Circular cake tin enlargement:

Original radius: 10 cm, new radius: 15 cm.

Area scales with square of linear factor:

Linear scaling factor: $\frac{30}{20} = 1.5$.

Area scaling factor: $1.5^2 = 2.25$.

New quantities:

- Flour: $250 \times 2.25 = 562.5$ g
- Sugar: $200 \times 2.25 = 450$ g
- Eggs: $4 \times 2.25 = 9$ eggs

8. Rectangular pizzas:

(a) Areas:

Small pizza: $20 \times 15 = 300 \text{ cm}^2$.

Large pizza: $35 \times 25 = 875 \text{ cm}^2$.

(b) Scaling factor: $\frac{875}{300} = \frac{35}{12} \approx 2.9167$.

New quantities:

- Cheese: $200 \times \frac{35}{12} \approx 583.33$ g
- Flour: $150 \times \frac{35}{12} = 437.5$ g
- Tomato sauce: $100 \times \frac{35}{12} \approx 291.67$ ml

9. Cake perimeter decoration:

(a) Perimeter: $2 \times (28 + 18) = 92$ cm.

(b) Sprinkles needed: $\frac{92}{4} = 23$ g.

(c) Double perimeter means linear dimensions multiplied by 2 (since perimeter is linear).

Area scales by $2^2 = 4$, so cake base and icing should be multiplied by 4.

Perimeter scales by 2, so sprinkles multiplied by 2.

10. Triangular dessert scaling:

(a) Original perimeter: $12 + 12 + 8 = 32$ cm.

(b) New perimeter: $32 \times 1.5 = 48$ cm.

(c) Sugar (for area): scales with square of linear factor.

Scaling factor: $1.5^2 = 2.25$.

Sugar: $120 \times 2.25 = 270$ g.

(d) Cocoa powder (for perimeter): scales linearly.

Cocoa: $80 \times 1.5 = 120$ g.

(e) Eggs: typically scale with volume (cube of linear factor), but recipe may vary.

If eggs scale with volume: $3 \times 1.5^3 = 3 \times 3.375 = 10.125$ eggs.

If eggs scale with area: $3 \times 2.25 = 6.75$ eggs.

Usually in baking, eggs scale with area for cakes, but problem doesn't specify.

Assuming linear scaling for simplicity: $3 \times 1.5 = 4.5$ eggs.

Best Buys Past

11. Cereal boxes:

500g box: $\frac{4.50}{500} = 0.009$ £/g = 9p per 10g.

750g box: $\frac{6.30}{750} = 0.0084$ £/g = 8.4p per 10g.

750g box is better value.

12. Soda cans:

Store A: 6 cans for \$4.80 $\Rightarrow$ per can: $\frac{4.80}{6} = 0.80$ dollars.

Store B: Single can normally \$0.95, 15% discount: $0.95 \times 0.85 = 0.8075$ dollars.

For 6 cans: Store A: \$4.80, Store B: $6 \times 0.8075 = 4.845$ dollars.

Store A is cheaper.

13. Juice bottles:

2L bottle: $\frac{3.20}{2} = 1.60$ €/L.

1.5L bottle: Sale price: $2.80 \times 0.80 = 2.24$ €.

Per liter: $\frac{2.24}{1.5} \approx 1.493$ €/L.

1.5L bottle on sale is better value.

14. Electronics store:

Headphones after discount: $120 \times 0.80 = 96$ £.

Speakers after discount: $85 \times 0.70 = 59.50$ £.

Buy 3 get cheapest free: Buy headphones, speakers, and a third cheap item (e.g., £10 item).

Total before free: $96 + 59.50 + 10 = 165.50$ £. Cheapest free: remove £10.

Pay: 155.50 £.

Selling to local shop at 50% of non-discounted price:

Headphones: $120 \times 0.50 = 60$ £, Speakers: $85 \times 0.50 = 42.50$ £.

Total if bought at discount and sold: $60 + 42.50 = 102.50$ £, cost 155.50 £, net loss.

Cheapest is to buy separately: $96 + 59.50 = 155.50$ £.

15. Furniture payment plans:

Plan A: Down: $0.25 \times 1200 = 300$ £, then $12 \times 80 = 960$ £. Total: 1260 £.

Plan B: $18 \times 75 = 1350$ £.

Plan A is better by $1350 - 1260 = 90$ £.

Percentage of original price: $\frac{90}{1200} \times 100\% = 7.5\%$.

Best Buys Future

16. Garden fencing:

Perimeter: $2 \times (10 + 8) = 36$ m.

Store A: $36 \times 12 = 432$ dollars.

Store B: Each section 2 m, need $\frac{36}{2} = 18$ sections.

Cost: $18 \times 20 = 360$ dollars.

Store B is cheaper.

17. Tiling floor:

Room area: $4 \times 5 = 20$ m².

Type A: $20 \times 25 = 500$ £.

Type B: Needs 15% more tiles: $20 \times 1.15 = 23$ m².

Cost: $23 \times 18 = 414$ £.

Type B is cheaper.

18. Paint coverage:

Total wall area: $(4 \times 3) + (5 \times 3) = 12 + 15 = 27$ m².

Standard paint: Liters needed: $\frac{27}{12} = 2.25$ L. Cost: $2.25 \times 35 = 78.75$ £.

Brand X: Coverage per liter: $12 \times 1.20 = 14.4$ m²/L.

Liters needed: $\frac{27}{14.4} = 1.875$ L. Cost: $1.875 \times 42 = 78.75$ £.

Same cost. Buying cheapest: 78.75 £.

19. **Grass seed:**

Lawn area: Rectangle: $10 \times 6 = 60 \text{ m}^2$.

Semicircle: $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 3^2 = \frac{9\pi}{2} \approx 14.137 \text{ m}^2$.

Total area: $60 + 14.137 = 74.137 \text{ m}^2$.

Seed A: kg needed: $\frac{74.137}{15} \approx 4.9425 \text{ kg}$. Cost: $4.9425 \times 4.80 \approx 23.724 \text{ dollars}$.

Seed B: kg needed: $\frac{74.137}{25} \approx 2.9655 \text{ kg}$. Cost: $2.9655 \times 7.50 \approx 22.241 \text{ dollars}$.

Seed B is more economical.

20. **Pool cover:**

Pool area: Rectangle: $8 \times 4 = 32 \text{ m}^2$.

Two semicircles = one circle: $\pi r^2 = \pi \times 2^2 = 4\pi \approx 12.566 \text{ m}^2$.

Total area: $32 + 12.566 = 44.566 \text{ m}^2$.

Material X: Cost: $44.566 \times 18 \approx 802.19 \text{ dollars}$. Per year: $\frac{802.19}{3} \approx 267.40 \text{ dollars/year}$.

Material Y: Cost: $44.566 \times 25 \approx 1114.15 \text{ dollars}$. Per year: $\frac{1114.15}{5} \approx 222.83 \text{ dollars/year}$.

Material Y is more economical per year. Cost per year: 222.83 dollars.

Scaling Past

21. **Drawing scaled up 150%:**

New length: $8 \times 1.5 = 12 \text{ cm}$.

22. **Map scale 1:25,000:**

1 cm represents $25,000 \text{ cm} = 250 \text{ m} = 0.25 \text{ km}$.

Percentage: $\frac{1}{25000} \times 100\% = 0.004\%$.

23. **Photo reduced to 80%:**

Reduced width = $12 \text{ cm} = 80\%$ of original.

Original width: $\frac{12}{0.8} = 15 \text{ cm}$.

24. **Bedroom scale drawing:**

Scale 2% of actual size $\Rightarrow$ ratio $2 : 100 = 1 : 50$.

Bed length: $2 \text{ m} = 200 \text{ cm}$. Drawn length: $200 \times 0.02 = 4 \text{ cm}$.

25. **Scale models:**

(a) Window width $1.2 \text{ m} = 120 \text{ cm}$.

Model A: $120 \times 0.025 = 3 \text{ cm}$.

Model B: $120 \times 0.04 = 4.8 \text{ cm}$.

(b) Model A height $45 \text{ cm} = 0.025$ of actual.

Actual height: $\frac{45}{0.025} = 1800 \text{ cm} = 18 \text{ m}$.

(c) Model B scale factor relative to Model A: $\frac{0.04}{0.025} = 1.6$.

Percentage larger: $(1.6 - 1) \times 100\% = 60\%$.

Scaling Future

26. **Square enlargement:**

Original perimeter 12 cm, side length 3 cm.

Scale factor 3, new side: 9 cm, new perimeter: $4 \times 9 = 36$ cm.

27. **Rectangular field:**

Scale 1:200, drawing dimensions 6 cm $\times$ 4 cm.

Actual dimensions: $6 \times 200 = 1200$ cm = 12 m, $4 \times 200 = 800$ cm = 8 m.

Actual area: $12 \times 8 = 96$ m².

28. **Equilateral triangle park:**

Scale 1:50, drawn side 2 cm.

Actual side: $2 \times 50 = 100$ cm = 1 m.