

Worked Solutions

Challenge: Decimals, Percentages, and Fractions

Q1. Write the following decimals as a fraction in simplest form and as a percentage:

$$(a) \ 0.125 = \frac{125}{1000} = \frac{1}{8}, \text{ and } 0.125 = 12.5\% \quad (b) \ 0.203125 = \frac{203125}{1000000} = \frac{13}{64}, \text{ and } 0.203125 = 20.3125\%.$$

Q2. Convert the following fractions into decimals and percentages:

$$(a) \ \frac{7}{20} = \frac{35}{100} = 0.35 = 35\% \quad (b) \ \frac{33}{320} = 33(0.003125) = 0.103125 = 10.3125\%.$$

Q3. Express the following percentages as decimals and fractions in simplest form:

$$(a) \ 17.5\% = 0.175 = \frac{175}{1000} = \frac{7}{40} \quad (b) \ 57.8125\% = \frac{57.8125}{100} = \frac{578125}{1000000} = \frac{37}{64}.$$

Q4. Basis points. Recall that 1 bp = 0.01%.

$$(a) \ 1 \text{ bp} = 0.01\% = 0.0001 = \frac{1}{10000} \quad (b) \ 175 \text{ bp} = 1.75\% = 0.0175 = \frac{175}{10000} = \frac{7}{400}.$$

Gilts. Treat the quoted return as the annual coupon rate on the amount borrowed.

$$(c) \ 5.678\% = 0.05678 = \frac{5678}{100000} = \frac{2839}{50000}.$$

(d) Annual coupon = $5.678\% \times 1000 = 0.05678(1000) = 56.78$. Over 30 years, coupons total $30(56.78) = 1703.40$. Including repayment of the original £1000:

$$1000 + 1703.40 = 2703.40.$$

So the total cost is £2703.40.

(e) $43.5 \text{ bp} = 0.435\% = 0.00435$. The coupon rises by $0.00435(1000) = 4.35$ per year. Over 30 years:

$$30(4.35) = 130.50.$$

So the extra cost is £130.50.

(f) An investor can achieve compounding by reinvesting every coupon payment in further gilts as soon as it is received, and by reinvesting maturing principal too. The new gilts themselves pay coupons, so interest is then earned on previous interest.

Q5. Below is the table of gilt yields. Using the table's labels, the 5-year period ends at the end of Year 4.

- (a) The yields increase with maturity:

$$4.377\% < 4.474\% < 4.488\% < 4.492\% < 4.544\%.$$

So the yield curve is upward-sloping.

- (b) The 5-year gilt has the highest yield, so it gives the best simple return if a single gilt is held to maturity; it is also the best starting point for a reinvestment strategy.
- (c) Use a coupon-reinvestment ladder. Start with the 5-year gilt. Each year, reinvest all coupon cash received at the previous year-end in the longest-dated gilt that still matures by the end of Year 4.
- (d) All coupon payments are rounded down to the nearest pence.

Start	Gilt bought	Matures	Yield	Amount	Annual coupon
0	5-year	end of Year 4	4.544%	£10,000.00	£454.40
1	4-year	end of Year 4	4.492%	£454.40	£20.41
2	3-year	end of Year 4	4.488%	£474.81	£21.30
3	2-year	end of Year 4	4.474%	£496.11	£22.19
4	1-year	end of Year 4	4.377%	£518.30	£22.68

The coupon cash reinvested at the start of each year is:

$$454.40, \quad 454.40+20.41 = 474.81, \quad 454.40+20.41+21.30 = 496.11, \quad 454.40+20.41+21.30+22.19 = 518.30$$

At the end of Year 4, every gilt has matured. The maturity payments are:

Gilt	Maturity payment
5-year	$10000.00 + 454.40 = 10454.40$
4-year	$454.40 + 20.41 = 474.81$
3-year	$474.81 + 21.30 = 496.11$
2-year	$496.11 + 22.19 = 518.30$
1-year	$518.30 + 22.68 = 540.98$

Thus

$$10454.40 + 474.81 + 496.11 + 518.30 + 540.98 = 12484.60,$$

so the total value at the end of the fifth year is £12,484.60.

- (e) Let r_s be the annual simple rate. Then

$$10000(1 + 5r_s) = 12484.60$$

so

$$r_s = \frac{12484.60 - 10000}{10000 \times 5} = \frac{2484.60}{50000} = 0.049692.$$

Thus the annual simple rate is 4.9692%.

(f) Let r_c be the annual compound rate. Then

$$10000(1 + r_c)^5 = 12484.60$$

so

$$r_c = \left(\frac{12484.60}{10000} \right)^{1/5} - 1 = 1.24846^{1/5} - 1 \approx 0.045382.$$

Thus the savings account would need to pay about 4.538% per year compounded annually.

(g) Let r_m be the monthly rate. Over 60 months,

$$10000(1 + r_m)^{60} = 12484.60$$

so

$$r_m = 1.24846^{1/60} - 1 \approx 0.003705.$$

Thus the account would need to pay about 0.3705% at the end of each month.