

Answer Key

Iterative Processes — Tables of Values and n th Terms

Section A

A1. Table: $U_1, \dots, U_6 = 4, 11, 18, 25, 32, 39$. $U_n = 7n - 3$.

A2. Table: $V_1, \dots, V_5 = 2, 6, 18, 54, 162$. $V_k = 2 \cdot 3^{k-1}$.

A3. Table: $F_0, \dots, F_6 = 72, 63, 54, 45, 36, 27, 18$. The first negative term is $F_9 = -9$, so the first negative value occurs at $j = 9$.

A4. Table: $P_1, \dots, P_5 = 1024, 256, 64, 16, 4$. $P_n = 4^{6-n} = 4096 \left(\frac{1}{4}\right)^n$.

Section B

B1. Values: $5, 8, 11, 14, 17, \dots$ $U_n = 3n + 2$.

B2. Values: $80, 40, 20, 10, 5, \dots$ $F_j = 80 \left(\frac{1}{2}\right)^{j-1}$.

B3. Recurrence: $V_{n+1} = 0.85V_n$, $V_0 = 200$. Values ($n = 0$ to 4 , 2 d.p.): $200.00, 170.00, 144.50, 122.83, 104.39$. $V_n = 200(0.85)^n$.

B4. Recurrence: $Q_{n+1} = \frac{Q_n}{2} + 6$, $Q_1 = 100$. Values: $100.00, 56.00, 34.00, 23.00, 17.50$. This sequence is neither arithmetic nor geometric; it has no simple formula from the methods above and approaches 12.

Section C

C1. $V_0 = 600$. $V_{k+1} = 1.05V_k$, $V_0 = 600$. Table ($k = 0$ to 4): $\pounds 600.00, \pounds 630.00, \pounds 661.50, \pounds 694.58, \pounds 729.31$. $V_k = 600(1.05)^k$. $V_{10} \approx \pounds 977.34$.

C2. Jess: $H_{n+1} = 0.75H_n$, $H_1 = 3$. Kwame: $H_{n+1} = 0.75H_n$, $H_0 = 4$. They use the same recurrence; they differ in starting index and starting value. Both give $H_3 = 1.6875$ m. Kwame's definition is slightly more natural because the original drop is H_0 and the formula works for all $n \geq 0$.

Section D

D1. Working backwards: $U_4 = 24$, $U_3 = 12$, $U_2 = 6$, $U_1 = 3$. $U_n = 3 \cdot 2^{n-1}$. Check: $U_5 = 3 \cdot 2^4 = 48$.

D2. Table ($j = 1$ to 8 , 2 d.p.): $100.00, 56.00, 34.00, 23.00, 17.50, 14.75, 13.38, 12.69$. The sequence approaches 12. Fixed point: $L = \frac{L}{2} + 6$, so $L = 12$. No finite term equals 12 because

$$F_j - 12 = \frac{88}{2^{j-1}} \neq 0$$

for any finite j .