

Worked Solutions: Conversions and Basis Points

Question 1

Write the following decimals as a fraction in simplest form and as a percentage.

(a) 0.125

Solution:

$$\begin{aligned} 0.125 &= \frac{125}{1000} \\ &= \frac{125 \div 125}{1000 \div 125} \\ &= \frac{1}{8} \end{aligned}$$

To convert to percentage, multiply by 100:

$$0.125 \times 100\% = 12.5\%$$

So, $0.125 = \frac{1}{8} = 12.5\%$.

(b) 0.203125

Solution:

$$\begin{aligned} 0.203125 &= \frac{203125}{1000000} \\ &= \frac{203125 \div 15625}{1000000 \div 15625} \\ &= \frac{13}{64} \end{aligned}$$

Note: $15625 = 5^6$, and $203125 \div 15625 = 13$.

To convert to percentage:

$$0.203125 \times 100\% = 20.3125\%$$

So, $0.203125 = \frac{13}{64} = 20.3125\%$.

Question 2

Convert the following fractions into decimals and percentages.

(a) $\frac{7}{20}$

Solution:

$$\begin{aligned}\frac{7}{20} &= 7 \div 20 = 0.35 \\ 0.35 \times 100\% &= 35\%\end{aligned}$$

So, $\frac{7}{20} = 0.35 = 35\%$.

(b) $\frac{33}{320}$

Solution:

$$\begin{aligned}\frac{33}{320} &= 33 \div 320 = 0.103125 \\ 0.103125 \times 100\% &= 10.3125\%\end{aligned}$$

So, $\frac{33}{320} = 0.103125 = 10.3125\%$.

Question 3

Express the following percentages as decimals and fractions in simplest form.

(a) 17.5%

Solution:

$$\begin{aligned}17.5\% &= \frac{17.5}{100} = 0.175 \\ 0.175 &= \frac{175}{1000} = \frac{7}{40}\end{aligned}$$

So, $17.5\% = 0.175 = \frac{7}{40}$.

(b) 57.8125%

Solution:

$$\begin{aligned}57.8125\% &= \frac{57.8125}{100} = 0.578125 \\ 0.578125 &= \frac{578125}{1000000} = \frac{37}{64}\end{aligned}$$

Note: $578125 \div 15625 = 37$ and $1000000 \div 15625 = 64$.

So, $57.8125\% = 0.578125 = \frac{37}{64}$.

Question 4

Basis points are units used in finance: 1 basis point = 0.01%.

- (a) Write one basis point as a decimal and a fraction in simplest form.

Solution:

$$\begin{aligned} 1 \text{ basis point} &= 0.01\% \\ &= \frac{0.01}{100} = 0.0001 \\ &= \frac{1}{10000} \end{aligned}$$

So, 1 basis point = $0.0001 = \frac{1}{10000}$.

- (b) Write 175 basis points as a decimal and a fraction in simplest form.

Solution:

$$\begin{aligned} 175 \text{ basis points} &= 175 \times 0.01\% = 1.75\% \\ &= \frac{1.75}{100} = 0.0175 \\ &= \frac{175}{10000} = \frac{7}{400} \end{aligned}$$

So, 175 basis points = $0.0175 = \frac{7}{400}$.

Gilts are UK government bonds with a coupon payment equal to a percentage of the amount borrowed.

- (c) Find 5.355% as a decimal and a fraction in its simplest form.

Solution:

$$\begin{aligned} 5.355\% &= \frac{5.355}{100} = 0.05355 \\ &= \frac{5355}{100000} = \frac{1071}{20000} \end{aligned}$$

So, $5.355\% = 0.05355 = \frac{1071}{20000}$.

- (d) Calculate the total cost for the government to borrow £1000 over 30 years, including payback of the original amount.

Solution:

$$\begin{aligned} \text{Annual coupon} &= 5.355\% \text{ of } 1000 \\ &= 0.05355 \times 1000 = 53.55 \\ \text{Total coupon payments over 30 years} &= 30 \times 53.55 = 1606.50 \\ \text{Repayment of principal} &= 1000 \\ \text{Total cost} &= 1606.50 + 1000 = 2606.50 \end{aligned}$$

So, the total cost is £2606.50.

- (e) Suppose the coupon payment of a 30 year gilt rises 43.5 basis points. How much more does it cost the government to borrow £1000 over 30 years?

Solution:

$$43.5 \text{ basis points} = 43.5 \times 0.01\% = 0.435\%$$

$$\text{Increase in annual coupon} = 0.435\% \text{ of } 1000$$

$$= 0.00435 \times 1000 = 4.35$$

$$\text{Additional cost over 30 years} = 30 \times 4.35 = 130.50$$

So, it costs an additional £130.50.