

Vectors, Translations, and Meaning

Worked Solutions

Extension Sheet

1 Vector translation review

Translating a point by $\begin{pmatrix} a \\ b \end{pmatrix}$ means: add a to the x -coordinate and b to the y -coordinate.

1. For $A = (2, 3)$ translated by $\mathbf{t} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$,

$$A' = (2, 3) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = (2 + 4, 3 + 1) = (6, 4).$$

So the destination is $\boxed{(6, 4)}$.

2. For $B = (-1, 5)$ translated by the same vector,

$$B' = (-1, 5) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = (-1 + 4, 5 + 1) = (3, 6).$$

So the destination is $\boxed{(3, 6)}$.

2 Word embeddings

Here $\overrightarrow{X Y}$ means the vector from the first word to the second word, so we compute it as $Y - X$.

From the grid:

$$\text{man} = (2, 5), \quad \text{woman} = (3, 8), \quad \text{king} = (7, 6), \quad \text{queen} = (8, 9),$$

$$\text{prince} = (5, 6), \quad \text{duke} = (4, 0), \quad \text{duchess} = (5, 3).$$

1.

$$\overrightarrow{\text{king} \quad \text{queen}} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

2.

$$\overrightarrow{\text{duke} \quad \text{duchess}} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

3. The vector $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is the same in both cases:

$$\text{king} \rightarrow \text{queen} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \text{duke} \rightarrow \text{duchess} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

It also maps man to woman. Therefore, in this embedding, $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ represents the **male-to-female** direction, i.e. a feminine/gender-change vector.

4. Since

$$\overrightarrow{\text{man} \quad \text{woman}} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix},$$

yes, it matches the answer to Question 3 exactly.

5. The missing word is at

$$(5, 6) + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = (6, 9).$$

Since (5, 6) is prince, applying the male-to-female vector gives

$$\text{prince} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \text{princess}.$$

So the missing word is princess.

3 Translating English to Chinese

First read the known coordinates from the grid.

English coordinates:

$$\text{man} = (9, 8), \quad \text{woman} = (10, 11), \quad \text{boy} = (7, 8), \quad \text{girl} = (8, 11),$$

$$\text{king} = (8, 6), \quad \text{queen} = (9, 9), \quad \text{prince} = (6, 6), \quad \text{princess} = (7, 9).$$

Chinese coordinates:

$$\text{男人} = (-6, -2), \quad \text{女人} = (-5, 1), \quad \text{男孩} = (-8, -2), \quad \text{女孩} = (-7, 1),$$

$$\text{国王} = (-7, -4), \quad \text{王妃} = (-6, -1), \quad \text{王子} = (-9, -4), \quad \text{公主} = (-9, 0).$$

1. Using the matching pair

$$\text{man} \rightarrow \text{男人},$$

the translation vector from English to Chinese is

$$\mathbf{t} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} - \begin{pmatrix} 9 \\ 8 \end{pmatrix} = \boxed{\begin{pmatrix} -15 \\ -10 \end{pmatrix}}.$$

Check with another pair:

$$\text{woman} + \mathbf{t} = \begin{pmatrix} 10 \\ 11 \end{pmatrix} + \begin{pmatrix} -15 \\ -10 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix},$$

which is indeed 女人.

2. The completed table is:

English	English coords	Chinese	Chinese coords
man	(9,8)	男人	(-6,-2)
woman	(10,11)	女人	(-5,1)
boy	(7,8)	男孩	(-8,-2)
girl	(8,11)	女孩	(-7,1)
king	(8,6)	国王	(-7,-4)
queen	(9,9)	王妃	(-6,-1)
prince	(6,6)	王子	(-9,-4)
princess	(7,9)	公主	(-9,0)

So the missing labels on the coordinate grid are:

$$(10, 11) = \text{woman}, \quad (9, 9) = \text{queen}, \quad (6, 6) = \text{prince}, \quad (7, 9) = \text{princess},$$

and on the Chinese side:

$$(-5, 1) = \text{女人}, \quad (-8, -2) = \text{男孩}, \quad (-9, 0) = \text{公主}.$$

3. The vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ appears between these English pairs:

$$\text{man} \rightarrow \text{boy}, \quad \text{woman} \rightarrow \text{girl}, \quad \text{king} \rightarrow \text{prince}, \quad \text{queen} \rightarrow \text{princess}.$$

In each case, the x -coordinate decreases by 2 and the y -coordinate stays the same. Thus $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ represents the meaning “**adult/older**” to “**younger/child**”, or simply the **age/youth direction** in this embedding.

4. The bottom row does not fit the pattern because English princess is the feminine form of prince, but Chinese 公主 is not a regular feminine form built from 王子.

Applying the translation vector to princess gives

$$\text{princess} + \mathbf{t} = \begin{pmatrix} 7 \\ 9 \end{pmatrix} + \begin{pmatrix} -15 \\ -10 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \end{pmatrix},$$

but the actual position of 公主 is

$$\begin{pmatrix} -9 \\ 0 \end{pmatrix}.$$

The difference is

$$\begin{pmatrix} -9 \\ 0 \end{pmatrix} - \begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

The Chinese characters explain why: words such as 女人, 女孩, 王妃 all contain the female radical 女, but 公主 does not. Chinese has a separate, lexicalised word for “princess” rather than constructing it as a regular feminine version of 王子. Therefore the regular translation-vector pattern breaks for this row.