

# Triangles Review — Worked Solutions

1. By Pythagoras' theorem,

$$x^2 = 4^2 + 3^2 = 16 + 9 = 25,$$

so

$$x = \sqrt{25} = 5.$$

2. Angles in a triangle sum to  $180^\circ$ :

$$52^\circ + 71^\circ + \theta = 180^\circ$$

$$\theta = 180^\circ - 52^\circ - 71^\circ = 57^\circ.$$

3. In the right triangle,

$$\tan 35^\circ = \frac{x}{7}.$$

Hence

$$x = 7 \tan 35^\circ \approx 4.90.$$

4. Using the cosine rule with the given labels:

$$3^2 = 5^2 + x^2 - 2(5)(x) \cos 40^\circ.$$

So

$$x^2 - 10x \cos 40^\circ + 16 = 0.$$

The discriminant is

$$D = (10 \cos 40^\circ)^2 - 64 < 0,$$

so there is no real value of  $x$ . The given data are inconsistent; no such triangle exists.

5. The hypotenuse is 7, so

$$x^2 + 3^2 = 7^2$$

$$x^2 = 49 - 9 = 40.$$

Thus

$$x = \sqrt{40} = 2\sqrt{10} \approx 6.32.$$

6. By Pythagoras' theorem,

$$x^2 = 5^2 + 2^2 = 25 + 4 = 29,$$

so

$$x = \sqrt{29} \approx 5.39.$$

7. Using the cosine rule,

$$6^2 = 4^2 + x^2 - 2(4)(x) \cos 55^\circ.$$

So

$$x^2 - 8x \cos 55^\circ - 20 = 0.$$

The positive solution is

$$x = \frac{8 \cos 55^\circ + \sqrt{64 \cos^2 55^\circ + 80}}{2} \approx 7.32.$$

8. The side  $x$  is opposite the  $40^\circ$  angle, so by the cosine rule,

$$x^2 = 3^2 + 4^2 - 2(3)(4) \cos 40^\circ$$

$$x^2 = 25 - 24 \cos 40^\circ.$$

Therefore

$$x = \sqrt{25 - 24 \cos 40^\circ} \approx 2.57.$$

9. By the cosine rule,

$$5^2 = 4^2 + 6^2 - 2(4)(6) \cos \theta.$$

Thus

$$25 = 52 - 48 \cos \theta$$

$$48 \cos \theta = 27$$

$$\cos \theta = \frac{27}{48} = \frac{9}{16}.$$

So

$$\theta = \arccos\left(\frac{9}{16}\right) \approx 55.77^\circ.$$

10. Here  $AB = 4$ ,  $AC = BC = 3$ , and  $\theta = \angle ACB$ .

$$4^2 = 3^2 + 3^2 - 2(3)(3) \cos \theta$$

$$16 = 18 - 18 \cos \theta$$

$$18 \cos \theta = 2$$

$$\cos \theta = \frac{1}{9}.$$

Therefore

$$\theta = \arccos\left(\frac{1}{9}\right) \approx 83.62^\circ.$$

11. The angle  $\theta = \angle ABC$  is opposite the side of length 4, so

$$4^2 = 3^2 + 6^2 - 2(3)(6) \cos \theta.$$

Thus

$$16 = 45 - 36 \cos \theta$$

$$36 \cos \theta = 29$$

$$\cos \theta = \frac{29}{36}.$$

So

$$\theta = \arccos\left(\frac{29}{36}\right) \approx 36.34^\circ.$$

12. In the right triangle,

$$\cos 32^\circ = \frac{6}{x},$$

so

$$x = \frac{6}{\cos 32^\circ} \approx 7.08.$$

13. Angles in a triangle sum to  $180^\circ$ :

$$40^\circ + 65^\circ + \theta = 180^\circ$$

$$\theta = 75^\circ.$$

14. The third angle is

$$180^\circ - 43^\circ - 34^\circ = 103^\circ.$$

By the sine rule,

$$\frac{x}{\sin 43^\circ} = \frac{5}{\sin 34^\circ}.$$

Hence

$$x = \frac{5 \sin 43^\circ}{\sin 34^\circ} \approx 6.10.$$

15. By Pythagoras' theorem,

$$x^2 = 4^2 + 2.5^2 = 16 + 6.25 = 22.25 = \frac{89}{4}.$$

Thus

$$x = \frac{\sqrt{89}}{2} \approx 4.72.$$

16. The angle  $\theta = \angle ACB$  is opposite  $AB = 4$ , with  $AC = 7$  and  $BC = 5$ :

$$4^2 = 7^2 + 5^2 - 2(7)(5) \cos \theta.$$

So

$$16 = 74 - 70 \cos \theta$$

$$70 \cos \theta = 58$$

$$\cos \theta = \frac{58}{70} = \frac{29}{35}.$$

Therefore

$$\theta = \arccos\left(\frac{29}{35}\right) \approx 34.05^\circ.$$

17. In the right triangle, relative to  $\theta$ , the opposite side is 8 and the adjacent side is 6:

$$\tan \theta = \frac{8}{6} = \frac{4}{3}.$$

So

$$\theta = \arctan\left(\frac{4}{3}\right) \approx 53.13^\circ.$$

18. Using the cosine rule,

$$6^2 = 5^2 + x^2 - 2(5)(x) \cos 45^\circ.$$

Since  $\cos 45^\circ = \frac{\sqrt{2}}{2}$ ,

$$36 = 25 + x^2 - 5\sqrt{2}x.$$

Thus

$$x^2 - 5\sqrt{2}x - 11 = 0.$$

Solving,

$$x = \frac{5\sqrt{2} + \sqrt{94}}{2} \approx 8.38.$$

19. The side  $x$  is opposite the  $40^\circ$  angle, and the hypotenuse is 10:

$$\sin 40^\circ = \frac{x}{10}.$$

Hence

$$x = 10 \sin 40^\circ \approx 6.43.$$

20. Angles in a triangle sum to  $180^\circ$ :

$$50^\circ + 70^\circ + \theta = 180^\circ$$

$$\theta = 60^\circ.$$