

Bearings Challenge — Answer Key

Introduction

1. Gold: 100 km from the start on bearing 123.5° , i.e. 83.38868 km east and 55.19370 km south.
2. Bob: due south 55.19370 km, then due east 83.38868 km.
3. Bob total distance: 138.58 km; time = 6.93 days.
4. Rob: 78.0557 km on bearing 135° , then east. Total = 106.35 km; time = 5.32 days; Rob arrives about 1 day 15 hours before Bob.
5. Best two-leg route: 50.00190 km on bearing 124° , then 50.00190 km on bearing 123° ; this takes less than a minute more than 5 days, so even with a 7-hour delay it beats Rob's time.

Challenge I: Integer Distances

1. (a) Bob, after rounding legs to whole km: 434 m from gold.
(b) Rob, after rounding legs to whole km: 237 m from gold.
2. Rounded quickest journey: 50 km on 124° , then 50 km on 123° ; endpoint is 99.99619 km from start and 3.81 m from gold, so not within 1 m; gold remains buried at this point.
3. Four-leg displacement: 0.5316 cm east and 91.3783 cm north.
4. Same route rotated 90° anticlockwise: 0.5316 cm north and 91.3783 cm west, i.e. -91.3783 cm east.
5. Use $A \approx -59868.41$, $B \approx -91604.77$; equivalently, 59868 reversed copies of the first 4-leg route and 91605 reversed copies of the rotated route. Total distance = 908838 km. This ends about 11 cm east and 37 cm north of the gold, about 39 cm away, so the gold is found.
6. Yes. For any target, real A, B can solve the two equations; rounding the repetition counts to integers leaves less than 50 cm error in each coordinate, hence the endpoint is within 1 m.

Challenge II: Two Legs Only

1. For the considered long first leg, the only return bearing that can get within 100 km of the start is the exact reverse bearing, e.g. 270° after 090° ; by symmetry, the only candidate return bearing is

$$XYZ^\circ = (ABC^\circ + 180^\circ) \bmod 360^\circ.$$

2. Necessary condition:

$$|M - N| \leq 100,$$

so N must lie in $[M - 100, M + 100]$.

3. Upper bound:

$$360 \cdot 6000 \cdot 360 \cdot 203 \approx 1.58 \times 10^{11}$$

possible destinations within 101 km.

4. Not necessarily, but possibly; checking all finite possibilities would be required.

5. Upper bound counts:

(a) $M \leq 100$: 2.66×10^9 .

(b) $100 < M \leq 500$: 5.26×10^9 (at most 180 return bearings).

(c) $500 < M \leq 2000$: 3.18×10^9 (at most 29 return bearings).

(d) $2000 < M \leq 6000$: 2.05×10^9 (at most 7 return bearings).

Total $\leq 1.32 \times 10^{10}$.

6. Reversing the leg order gives the same endpoint, so the bound halves:

$$U \leq 6.6 \times 10^9 < 10^{10}.$$

7. Not every location is reachable within 1 m: the reachable area is at most $6.6 \times 10^9 \pi \text{ m}^2$, less than the circle area $10^{10} \pi \text{ m}^2$; at least $3 \times 10^9 \pi \text{ m}^2$ is unreachable.

8. Naive brute force would take over 42 hours; the vectorised NumPy version takes about 40 minutes and finds 82 candidate two-leg journeys ending within 15 m of the gold. Plotting them shows none is within 1 m, so the gold cannot be found by a two-leg journey.

9. One two-leg journey ends within 2 m of the gold: 4937 km on bearing 004° , then 4987 km on bearing 183° ; final coordinates (83.38830, -55.19177).