

Recurring Decimals into Fractions

Answer Key

Part 1: Reading and Writing Recurring Decimals

Q1. First eight decimal places:

(a) $0.\dot{5} = 0.55555555$

(b) $0.\dot{1}\dot{8} = 0.18181818$

(c) $0.3\dot{7} = 0.37777777$

(d) $0.\dot{2}0\dot{6} = 0.20620620$

(e) $0.4\dot{5}\dot{1} = 0.45151515$

(f) $0.\dot{9} = 0.99999999$

Q2. Dot notation:

(a) $0.8888\dots = 0.\dot{8}$

(b) $0.636363\dots = 0.\dot{6}\dot{3}$

(c) $0.24444\dots = 0.2\dot{4}$

(d) $0.157157\dots = 0.\dot{1}5\dot{7}$

(e) $0.91666\dots = 0.91\dot{6}$

(f) $0.40505\dots = 0.4\dot{0}\dot{5}$

Q3.

(a) $0.4\dot{5}$ is bigger.

(b) One example: $0.\dot{3}\dot{5}$ lies between $0.\dot{3}$ and 0.4 .

Part 2: Finding Fractions from Recurring Decimals

Q4. $0.\dot{7} = \frac{7}{9}$

Q5. $0.\dot{8} = \frac{8}{9}$

Q6.

(a) $\frac{2}{9}$

(b) $\frac{5}{9}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

Part 3: Longer Repeating Blocks

Q7. $0.\dot{7}2 = \frac{8}{11}$

Q8.

(a) $\frac{5}{33}$

(b) $\frac{2}{33}$

(c) $\frac{6}{11}$

(d) $\frac{10}{11}$

(e) $\frac{4}{37}$

Q9.

(a) Multiply by 10^n , where n is the number of digits in the repeating block.

(b) $0.1\dot{2} = \frac{12}{99} = \frac{4}{33}$. Ben and Kayla are both correct; Ben has just simplified Kayla's fraction.

Part 4: When the Repeating Part Does Not Start Straight Away

Q10. $0.2\dot{7} = \frac{5}{18}$

Q11.

(a) $\frac{13}{30}$

(b) $\frac{7}{12}$

(c) $\frac{1}{12}$

(d) $\frac{41}{330}$

Q12.

(a) $\frac{25}{9}$

(b) $\frac{16}{11}$

(c) $\frac{35}{11}$

Q13.

- (a) $\frac{8}{9}$
- (b) $\frac{7}{11}$
- (c) $\frac{23}{90}$
- (d) $\frac{1}{37}$
- (e) $\frac{13}{18}$
- (f) $\frac{4}{33}$
- (g) $\frac{5}{12}$
- (h) $\frac{29}{450}$

Part 5: Challenges**Q14.**

- (a) $0.\dot{9} = 1$
- (b) Since $10x = 9.\dot{9}$, subtracting gives $9x = 9$, so $x = 1$. Also $3 \times 0.\dot{3} = 1$. There is no “just below” 1; between any number below 1 and 1 there is always another number.
- (c) No. For any $a < 1$, we have $a < \frac{a+1}{2} < 1$. So there is no largest number smaller than 1.

Q15.

- (a) $\frac{1}{2} = 0.5$, $\frac{1}{3} = 0.\dot{3}$, $\frac{1}{4} = 0.25$, $\frac{1}{5} = 0.2$, $\frac{1}{6} = 0.1\dot{6}$,
 $\frac{1}{7} = 0.14285\dot{7}$, $\frac{1}{8} = 0.125$, $\frac{1}{9} = 0.\dot{1}$, $\frac{1}{10} = 0.1$,
 $\frac{1}{11} = 0.\dot{0}\dot{9}$, $\frac{1}{12} = 0.08\dot{3}$, $\frac{1}{16} = 0.0625$.

Terminating decimals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{16}$.

- (b) The terminating denominators have only prime factors 2 and 5.
- (c) $\frac{1}{15}$ recurs, $\frac{1}{40}$ terminates, $\frac{1}{35}$ recurs, $\frac{1}{64}$ terminates.
- (d) A fraction in lowest terms terminates exactly when its denominator has no prime factors other than 2 and 5.

Q16.

$$\begin{aligned}
\text{(a)} \quad \frac{2}{7} &= 0.\dot{2}8571\dot{4}, \\
\frac{3}{7} &= 0.4\dot{2}857\dot{1}, \\
\frac{4}{7} &= 0.\dot{5}7142\dot{8}, \\
\frac{5}{7} &= 0.\dot{7}1428\dot{5}, \\
\frac{6}{7} &= 0.\dot{8}5714\dot{2}.
\end{aligned}$$

(b) They all use the same six digits in the same cyclic order:

$$1 \rightarrow 4 \rightarrow 2 \rightarrow 8 \rightarrow 5 \rightarrow 7 \rightarrow 1.$$

$$\text{(c)} \quad x = 0.\dot{2}8571\dot{4} = \frac{285714}{999999} = \frac{2}{7}.$$

Q17.

$$\text{(a)} \quad \text{One valid example is } 0.\dot{2}\dot{7} = \frac{3}{11}.$$

$$\text{(b)} \quad \frac{5}{6} = 0.8\dot{3}.$$

(c) Yes. If the repeating block has n digits and represents the number N , then

$$10^n x = N + x,$$

so

$$x = \frac{N}{10^n - 1}.$$

Non-repeating leading digits can first be shifted past by multiplying by a power of 10. Hence every recurring decimal is rational. Since π is irrational, no recurring decimal can be exactly equal to π .