

Averages: Mean & Median

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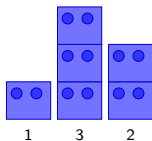
Questions and short answers

- ▶ Retrieve and Connect 1
- ▶ Retrieve and Connect 2
- ▶ Retrieve and Connect 3
- ▶ Retrieve and Connect 4
- ▶ Retrieve and Connect 5

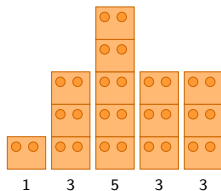
Worked solutions

- ▶ Retrieve and Connect 1
- ▶ Retrieve and Connect 2
- ▶ Retrieve and Connect 3
- ▶ Retrieve and Connect 4
- ▶ Retrieve and Connect 5

Retrieve and Connect



Q7: balance these



Q8: balance these

1. $12 \div 4 = ?$

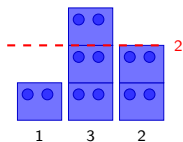
2. $18 \div 3 = ?$

3. $15 \div 3 = ?$

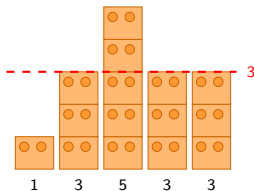
4. $28 \div 4 = ?$

- Four friends share 20 sweets equally. How many do they each get?
- Three children have 2, 5 and 8 stickers. They combine them and share them equally. How many does each child get?
- Draw a picture of the top towers (left), balanced to the same height without changing the total number of bricks.
- Draw a picture of the bottom towers (left), balanced to the same height without changing the total number of bricks.

Retrieve and Connect



Q7: balance these



Q8: balance these

1. $12 \div 4 = 3$

2. $18 \div 3 = 6$

3. $15 \div 3 = 5$

4. $28 \div 4 = 7$

5. Four friends share 20 sweets equally. How many do they each get? **5 sweets**
6. Three children have 2, 5 and 8 stickers. They combine them and share them equally. How many does each child get? **5 stickers**
7. Draw a picture of the top towers (left), balanced to the same height without changing the total number of bricks.
8. Draw a picture of the bottom towers (left), balanced to the same height without changing the total number of bricks.

Worked Solution: Retrieve and Connect 1, Question 1

Question: $12 \div 4 = ?$

Worked Solution: Since $4 \times 3 = 12$,

$$12 \div 4 = 3.$$

Worked Solution: Retrieve and Connect 1, Question 2

Question: $18 \div 3 = ?$

Worked Solution: Since $3 \times 6 = 18$,

$$18 \div 3 = 6.$$

Worked Solution: Retrieve and Connect 1, Question 3

Question: $15 \div 3 = ?$

Worked Solution: Since $3 \times 5 = 15$,

$$15 \div 3 = 5.$$

Worked Solution: Retrieve and Connect 1, Question 4

Question: $28 \div 4 = ?$

Worked Solution: Since $4 \times 7 = 28$,

$$28 \div 4 = 7.$$

Worked Solution: Retrieve and Connect 1, Question 5

Question: Four friends share 20 sweets equally. How many do they each get?

Worked Solution: Sharing equally means dividing the total by the number of friends:

$$20 \div 4 = 5.$$

Each friend gets 5 sweets.

Worked Solution: Retrieve and Connect 1, Question 6

Question: Three children have 2, 5 and 8 stickers. They combine them and share them equally. How many does each child get?

Worked Solution: Total stickers:

$$2 + 5 + 8 = 15.$$

There are 3 children, so each child gets

$$15 \div 3 = 5$$

stickers.

Worked Solution: Retrieve and Connect 1, Question 7

Question: Draw a picture of the top towers, balanced to the same height without changing the total number of bricks.

Worked Solution: The top towers have heights 1, 3, 2.

Total bricks:

$$1 + 3 + 2 = 6.$$

There are 3 towers, so the balanced height is

$$6 \div 3 = 2.$$

The picture should show all the bricks levelled out at height 2.

Worked Solution: Retrieve and Connect 1, Question 8

Question: Draw a picture of the bottom towers, balanced to the same height without changing the total number of bricks.

Worked Solution: The bottom towers have heights 1, 3, 5, 3, 3.
Total bricks:

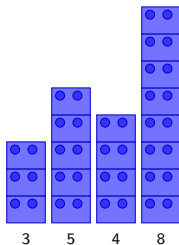
$$1 + 3 + 5 + 3 + 3 = 15.$$

There are 5 towers, so the balanced height is

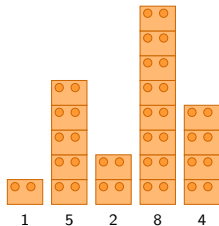
$$15 \div 5 = 3.$$

The picture should show all the bricks levelled out at height 3.

Retrieve and Connect



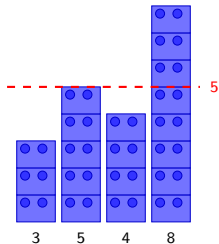
Q7: balance these



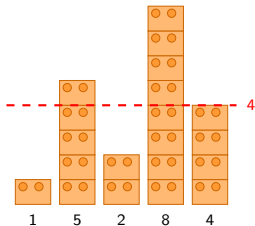
Q8: balance these?

1. $7 \times 8 = ?$
2. $63 \div 7 = ?$
3. $48 \div 8 = ?$
4. $5 \times 9 = ?$
5. Five children have 4, 7, 3, 6, 5 marbles. Shared equally, how many each?
6. Six friends earn £8, £14, £10, £6, £12, £10 at a car wash. Split fairly — how much each?
7. Eight runners finish in 54, 48, 61, 57, 50, 63, 55, 52 s. Without doing any calculations, estimate the average time.
8. What heights would the top towers balance out to?
9. What heights would the bottom towers balance out to?

Retrieve and Connect



Q7: balance these



Q8: balance these?

1. $7 \times 8 = 56$

2. $63 \div 7 = 9$

3. $48 \div 8 = 6$

4. $5 \times 9 = 45$

5. Five children have 4, 7, 3, 6, 5 marbles. Shared equally, how many each? **5 marbles**

6. Six friends earn £8, £14, £10, £6, £12, £10 at a car wash. Split fairly — how much each? **£10 each**

7. Eight runners finish in 54, 48, 61, 57, 50, 63, 55, 52 s. Without doing any calculations, estimate the average time. **about 55 s**

8. What heights would the top towers balance out to? **5 bricks high**

9. What heights would the bottom towers balance out to? **4 bricks high**

Worked Solution: Retrieve and Connect 2, Question 1

Question: $7 \times 8 = ?$

Worked Solution:

$$7 \times 8 = 56.$$

This is one of the times-table facts for 7 and 8.

Worked Solution: Retrieve and Connect 2, Question 2

Question: $63 \div 7 = ?$

Worked Solution: Since $7 \times 9 = 63$,

$$63 \div 7 = 9.$$

Worked Solution: Retrieve and Connect 2, Question 3

Question: $48 \div 8 = ?$

Worked Solution: Since $8 \times 6 = 48$,

$$48 \div 8 = 6.$$

Worked Solution: Retrieve and Connect 2, Question 4

Question: $5 \times 9 = ?$

Worked Solution:

$$5 \times 9 = 45,$$

because $9 \times 5 = 45$.

Worked Solution: Retrieve and Connect 2, Question 5

Question: Five children have 4, 7, 3, 6, 5 marbles. Shared equally, how many each?

Worked Solution: Total marbles:

$$4 + 7 + 3 + 6 + 5 = 25.$$

There are 5 children, so each child gets

$$25 \div 5 = 5$$

marbles.

Worked Solution: Retrieve and Connect 2, Question 6

Question: Six friends earn £8, £14, £10, £6, £12, £10 at a car wash. Split fairly — how much each?

Worked Solution: Total earned:

$$8 + 14 + 10 + 6 + 12 + 10 = 60$$

pounds. There are 6 friends, so each gets

$$60 \div 6 = 10$$

pounds, i.e. \$10 each.

Worked Solution: Retrieve and Connect 2, Question 7

Question: Eight runners finish in 54, 48, 61, 57, 50, 63, 55, 52 s. Without doing any calculations, estimate the average time.

Worked Solution: The times are spread fairly evenly around 55 s: some are a little above and some are a little below. So a good estimate is

about 55 seconds.

Checking exactly:

$$54 + 48 + 61 + 57 + 50 + 63 + 55 + 52 = 440,$$

so the mean is

$$440 \div 8 = 55 \text{ s.}$$

The estimate is exact.

Worked Solution: Retrieve and Connect 2, Question 8

Question: What heights would the top towers balance out to?

Worked Solution: The top towers have heights 3, 5, 4, 8.

Total bricks:

$$3 + 5 + 4 + 8 = 20.$$

There are 4 towers, so the balanced height is

$$20 \div 4 = 5$$

bricks high.

Worked Solution: Retrieve and Connect 2, Question 9

Question: What heights would the bottom towers balance out to?

Worked Solution: The bottom towers have heights 1, 5, 2, 8, 4.

Total bricks:

$$1 + 5 + 2 + 8 + 4 = 20.$$

There are 5 towers, so the balanced height is

$$20 \div 5 = 4$$

bricks high.

Retrieve and Connect

1. $3 + 4 + 7 = ?$
2. $15 + 27 + 18 = ?$
3. $60 \div 4 = ?$
4. $-15 \div 3 = ?$
5. What is the mean of: 4, 6, 2, 8
6. What is the mean of: 3, 9, 5, 7, 1
7. What is the mean of: -3, 5, -1, 7, 2
8. What is the mean of: -8, -2, -5, -1, -4
9. The mean of four numbers is 9. Three are 6, 11 and 8. Find the fourth.
10. The mean of 3, 7, x , 10 is 8. What is x ?

Retrieve and Connect

1. $3 + 4 + 7 = 14$
2. $15 + 27 + 18 = 60$
3. $60 \div 4 = 15$
4. $-15 \div 3 = -5$
5. What is the mean of: 4, 6, 2, 8 **5**
6. What is the mean of: 3, 9, 5, 7, 1 **5**
7. What is the mean of: -3, 5, -1, 7, 2 **2**
8. What is the mean of: -8, -2, -5, -1, -4 **-4**
9. The mean of four numbers is 9. Three are 6, 11 and 8. Find the fourth.
11
10. The mean of 3, 7, x , 10 is 8. What is x ? **12**

Worked Solution: Retrieve and Connect 3, Question 1

Question: $3 + 4 + 7 = ?$

Worked Solution:

$$3 + 4 + 7 = 14.$$

Add in stages: $3 + 4 = 7$, then $7 + 7 = 14$.

Worked Solution: Retrieve and Connect 3, Question 2

Question: $15 + 27 + 18 = ?$

Worked Solution:

$$15 + 27 + 18 = 60.$$

For example, $15 + 27 = 42$, and $42 + 18 = 60$.

Worked Solution: Retrieve and Connect 3, Question 3

Question: $60 \div 4 = ?$

Worked Solution: Since $4 \times 15 = 60$,

$$60 \div 4 = 15.$$

Worked Solution: Retrieve and Connect 3, Question 4

Question: $-15 \div 3 = ?$

Worked Solution: First $15 \div 3 = 5$. A negative number divided by a positive number is negative, so

$$-15 \div 3 = -5.$$

Worked Solution: Retrieve and Connect 3, Question 5

Question: What is the mean of 4, 6, 2, 8?

Worked Solution: Sum:

$$4 + 6 + 2 + 8 = 20.$$

There are 4 numbers, so

$$\text{mean} = \frac{20}{4} = 5.$$

Worked Solution: Retrieve and Connect 3, Question 6

Question: What is the mean of 3, 9, 5, 7, 1?

Worked Solution: Sum:

$$3 + 9 + 5 + 7 + 1 = 25.$$

There are 5 numbers, so

$$\text{mean} = \frac{25}{5} = 5.$$

Worked Solution: Retrieve and Connect 3, Question 7

Question: What is the mean of $-3, 5, -1, 7, 2$?

Worked Solution: Sum the numbers:

$$-3 + 5 - 1 + 7 + 2 = 10.$$

There are 5 numbers, so

$$\text{mean} = \frac{10}{5} = 2.$$

Worked Solution: Retrieve and Connect 3, Question 8

Question: What is the mean of $-8, -2, -5, -1, -4$?

Worked Solution: Sum the numbers:

$$-8 - 2 - 5 - 1 - 4 = -20.$$

There are 5 numbers, so

$$\text{mean} = \frac{-20}{5} = -4.$$

Worked Solution: Retrieve and Connect 3, Question 9

Question: The mean of four numbers is 9. Three are 6, 11 and 8. Find the fourth.

Worked Solution: If the mean of 4 numbers is 9, their total is

$$4 \times 9 = 36.$$

The three known numbers add to

$$6 + 11 + 8 = 25.$$

So the fourth number is

$$36 - 25 = 11.$$

Worked Solution: Retrieve and Connect 3, Question 10

Question: The mean of 3, 7, x , 10 is 8. What is x ?

Worked Solution: If the mean of 4 numbers is 8, their total is

$$4 \times 8 = 32.$$

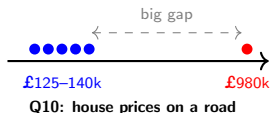
The known numbers add to

$$3 + 7 + 10 = 20.$$

Therefore

$$x = 32 - 20 = 12.$$

Retrieve and Connect



1. $4 + 2 + 10 = ?$
2. $-4 + 9 = ?$
3. $-3 - 5 = ?$
4. $6 \times 4 = ?$
5. Order smallest to largest: 8, 3, 11, 5, 7. Which is the middle value?
6. Order: 21, 14, 9, 35, 17, 28, 6. Which is the middle value?
7. For 3, 7, 8, 9, 41: *estimate* whether the middle value is above or below the mean. Then check.
8. What is the middle value of:
 $-7, -2, -5, 1, -1, 3, -3$
9. What is the middle value of: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{5}{8}$
10. House prices on a road are: £130k, £125k, £140k, £128k, £135k, £980k.
Why might the mean not be helpful?

Retrieve and Connect



- $4 + 2 + 10 = 16$
- $-4 + 9 = 5$
- $-3 - 5 = -8$
- $6 \times 4 = 24$
- Order smallest to largest: 8, 3, 11, 5, 7. Which is the middle value? 7
- Order: 21, 14, 9, 35, 17, 28, 6. Which is the middle value? 17
- For 3, 7, 8, 9, 41: *estimate* whether the middle value is above or below the mean. Then check.
below: median 8, mean 13.6
- What is the middle value of:
-7, -2, -5, 1, -1, 3, -3 -2
- What is the middle value of: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{5}{8}$ $\frac{1}{2}$
- House prices on a road are: £130k, £125k, £140k, £128k, £135k, £980k.
Why might the mean not be helpful? the £980k outlier pulls the mean up

Worked Solution: Retrieve and Connect 4, Question 1

Question: $4 + 2 + 10 = ?$

Worked Solution:

$$4 + 2 + 10 = 16.$$

Add in stages: $4 + 2 = 6$, then $6 + 10 = 16$.

Worked Solution: Retrieve and Connect 4, Question 2

Question: $-4 + 9 = ?$

Worked Solution: Starting at -4 and moving up 9 gives

$$-4 + 9 = 5.$$

Equivalently, $9 - 4 = 5$.

Worked Solution: Retrieve and Connect 4, Question 3

Question: $-3 - 5 = ?$

Worked Solution: Subtracting 5 from -3 means moving further down the number line:

$$-3 - 5 = -8.$$

Worked Solution: Retrieve and Connect 4, Question 4

Question: $6 \times 4 = ?$

Worked Solution:

$$6 \times 4 = 24,$$

because $4 + 4 + 4 + 4 + 4 + 4 = 24$.

Worked Solution: Retrieve and Connect 4, Question 5

Question: Order 8, 3, 11, 5, 7 smallest to largest. Which is the middle value?

Worked Solution: In order:

3, 5, 7, 8, 11.

There are 5 numbers, so the middle value is the 3rd number:

7.

Worked Solution: Retrieve and Connect 4, Question 6

Question: Order 21, 14, 9, 35, 17, 28, 6. Which is the middle value?

Worked Solution: In order:

6, 9, 14, 17, 21, 28, 35.

There are 7 numbers, so the middle value is the 4th number:

17.

Worked Solution: Retrieve and Connect 4, Question 7

Question: For 3, 7, 8, 9, 41: estimate whether the middle value is above or below the mean. Then check.

Worked Solution: The middle value, or median, is 8.

The mean is

$$\frac{3 + 7 + 8 + 9 + 41}{5} = \frac{68}{5} = 13.6.$$

So the middle value 8 is below the mean. The large value 41 pulls the mean upwards.

Worked Solution: Retrieve and Connect 4, Question 8

Question: What is the middle value of $-7, -2, -5, 1, -1, 3, -3$?

Worked Solution: Write the numbers in order:

$$-7, -5, -3, -2, -1, 1, 3.$$

There are 7 numbers, so the middle value is the 4th number:

$$-2.$$

Worked Solution: Retrieve and Connect 4, Question 9

Question: What is the middle value of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{5}{8}$?

Worked Solution: Write each fraction in eighths:

$$\frac{4}{8}, \frac{2}{8}, \frac{6}{8}, \frac{1}{8}, \frac{5}{8}.$$

In order:

$$\frac{1}{8}, \frac{2}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}.$$

The middle value is

$$\frac{4}{8} = \frac{1}{2}.$$

Worked Solution: Retrieve and Connect 4, Question 10

Question: House prices on a road are £130k, £125k, £140k, £128k, £135k, £980k. Why might the mean not be helpful?

Worked Solution: In thousands of pounds, the prices are

130, 125, 140, 128, 135, 980.

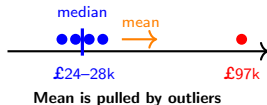
Their mean is

$$\frac{130 + 125 + 140 + 128 + 135 + 980}{6} = \frac{1638}{6} = 273.$$

That is \$273k, which is far above five of the six prices.

The \$980k price is an outlier: it pulls the mean up so it is not representative of the road. The median would be more helpful here.

Retrieve and Connect



1. $-6 + (-2) = ?$
2. $\frac{1}{3} + \frac{1}{2} = ?$
3. Median of: 3, 9, 4, 7, 1
4. Mean **and** median of: 5, 8, 3, 12, 7. Which is larger?
5. Five salaries are: £24k, £26k, £25k, £28k, **£97k**. Would you prefer to use the mean or the median to find the average?
6. For $-10, -1, 0, 2, 3$: *estimate* whether the mean is positive or negative. Then calculate both mean and median.
7. Find the mean and median of:
 $-6, -2, 4, -3, 7, 0$
8. What is the median of: $\frac{1}{3}, \frac{3}{4}, \frac{1}{2}, \frac{1}{6}, \frac{2}{3}, \frac{1}{4}$
9. True or False?
 - (a) The median is always one of the values in the dataset.
 - (b) The mean and median are always different.

Retrieve and Connect



1. $-6 + (-2) = -8$ 2. $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$
3. Median of: 3, 9, 4, 7, 1 4
4. Mean **and** median of: 5, 8, 3, 12, 7. Which is larger? **equal (both 7)**
5. Five salaries are: £24k, £26k, £25k, £28k, **£97k**. Would you prefer to use the mean or the median to find the average? **the median**
6. For $-10, -1, 0, 2, 3$: *estimate* whether the mean is positive or negative. Then calculate both mean and median. **negative: mean -1.2 ; median 0**
7. Find the mean and median of:
 $-6, -2, 4, -3, 7, 0$ **mean 0; median -1**
8. What is the median of: $\frac{1}{3}, \frac{3}{4}, \frac{1}{2}, \frac{1}{6}, \frac{2}{3}, \frac{1}{4}$ **$\frac{5}{12}$**
9. True or False?
(a) The median is always one of the values in the dataset. **False**
(b) The mean and median are always different. **False**

Worked Solution: Retrieve and Connect 5, Question 1

Question: $-6 + (-2) = ?$

Worked Solution: Adding -2 is the same as subtracting 2:

$$-6 + (-2) = -6 - 2 = -8.$$

Worked Solution: Retrieve and Connect 5, Question 2

Question: $\frac{1}{3} + \frac{1}{2} = ?$

Worked Solution: Use a common denominator of 6:

$$\frac{1}{3} = \frac{2}{6} \quad \text{and} \quad \frac{1}{2} = \frac{3}{6}.$$

So

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}.$$

Worked Solution: Retrieve and Connect 5, Question 3

Question: Median of 3, 9, 4, 7, 1?

Worked Solution: Write in order:

1, 3, 4, 7, 9.

There are 5 numbers, so the median is the middle value:

4.

Worked Solution: Retrieve and Connect 5, Question 4

Question: Mean and median of 5, 8, 3, 12, 7. Which is larger?

Worked Solution: In order:

3, 5, 7, 8, 12.

So the median is 7.

The mean is

$$\frac{5 + 8 + 3 + 12 + 7}{5} = \frac{35}{5} = 7.$$

They are equal: neither is larger.

Worked Solution: Retrieve and Connect 5, Question 5

Question: Five salaries are £24k, £26k, £25k, £28k, £97k. Would you prefer to use the mean or the median to find the average?

Worked Solution: Sort the salaries:

24, 25, 26, 28, 97

(in thousands of pounds).

The median is 26k.

The mean is pulled up by the large outlier:

$$\frac{24 + 25 + 26 + 28 + 97}{5} = \frac{200}{5} = 40\text{k}.$$

The median is more representative, so we should prefer the median.

Worked Solution: Retrieve and Connect 5, Question 6

Question: For $-10, -1, 0, 2, 3$: estimate whether the mean is positive or negative. Then calculate both mean and median.

Worked Solution: The negative numbers dominate: -10 and -1 together are -11 , while the positive numbers add to 5 . So the mean should be negative.

Mean:

$$\frac{-10 - 1 + 0 + 2 + 3}{5} = \frac{-6}{5} = -1.2.$$

In order:

$$-10, -1, 0, 2, 3.$$

The median is the middle value:

$$0.$$

Worked Solution: Retrieve and Connect 5, Question 7

Question: Find the mean and median of $-6, -2, 4, -3, 7, 0$.

Worked Solution: Mean:

$$\frac{-6 - 2 + 4 - 3 + 7 + 0}{6} = \frac{0}{6} = 0.$$

In order:

$$-6, -3, -2, 0, 4, 7.$$

There are 6 numbers, so the median is the mean of the two middle values:

$$\frac{-2 + 0}{2} = -1.$$

Worked Solution: Retrieve and Connect 5, Question 8

Question: What is the median of $\frac{1}{3}, \frac{3}{4}, \frac{1}{2}, \frac{1}{6}, \frac{2}{3}, \frac{1}{4}$?

Worked Solution: Write each fraction in twelfths:

$$\frac{1}{6} = \frac{2}{12}, \quad \frac{1}{4} = \frac{3}{12}, \quad \frac{1}{3} = \frac{4}{12}, \quad \frac{1}{2} = \frac{6}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}.$$

In order:

$$\frac{2}{12}, \quad \frac{3}{12}, \quad \frac{4}{12}, \quad \frac{6}{12}, \quad \frac{8}{12}, \quad \frac{9}{12}.$$

There are 6 numbers, so the median is halfway between the 3rd and 4th:

$$\frac{\frac{4}{12} + \frac{6}{12}}{2} = \frac{10}{12} \div 2 = \frac{5}{12}.$$

Worked Solution: Retrieve and Connect 5, Question 9

Question: True or False? (a) The median is always one of the values in the dataset. (b) The mean and median are always different. (c) A very large outlier affects the median more than the mean.

Worked Solution:

(a) **False.** For example, in 1, 2, 3, 4, the median is

$$\frac{2 + 3}{2} = 2.5,$$

which is not in the dataset.

(b) **False.** For example, in 5, 8, 3, 12, 7, both the mean and median are 7.

(c) **False.** A very large outlier affects the mean more than the median. The median is resistant to outliers.