

Algebraic Fractions: Starters

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Questions and short answers

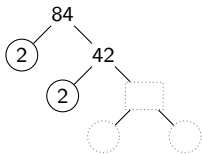
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Worked solutions

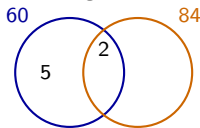
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Starter 1: Prime Factors & HCF

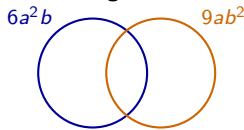
Factor tree



Venn diagram A



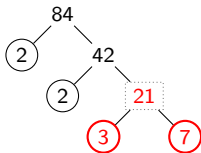
Venn diagram B



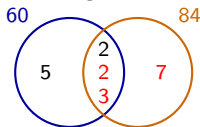
1. Simplify $\sqrt{20}$.
2. Work out 2^{-1} and $9^{\frac{1}{2}}$.
3. Complete the factor tree. Write 84 as a product of prime factors in index form.
4. $60 = 2^2 \times 3 \times 5$. Complete Venn diagram A for 60 and 84, then find the HCF and the LCM of 60 and 84.
5. Simplify $\frac{60}{84}$ by dividing by the HCF. Which numbers in Venn diagram A are not in the HCF?
6. Simplify $\frac{x^5}{x^2}$ by writing out the xs and cancelling.
7. (Challenge) Complete Venn diagram B for $6a^2b$ and $9ab^2$. Use it to simplify $\frac{6a^2b}{9ab^2}$.

Starter 1: Prime Factors & HCF

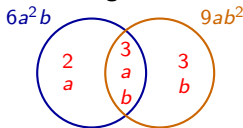
Factor tree



Venn diagram A



Venn diagram B



1. Simplify $\sqrt{20}$.
 $\sqrt{4 \times 5} = 2\sqrt{5}$
2. Work out 2^{-1} and $9^{\frac{1}{2}}$.
 $\frac{1}{2}$ and 3
3. Complete the factor tree. Write 84 as a product of prime factors in index form. $84 = 2^2 \times 3 \times 7$
4. $60 = 2^2 \times 3 \times 5$. Complete Venn diagram A for 60 and 84, then find the HCF and the LCM of 60 and 84. **HCF = 12; LCM = $2^2 \times 3 \times 5 \times 7 = 420$**
5. Simplify $\frac{60}{84}$ by dividing by the HCF. Which numbers in Venn diagram A are not in the HCF?
 $\frac{5}{7}$: the 5 and the 7, which are not shared
6. Simplify $\frac{x^5}{x^2}$ by writing out the xs and cancelling.
 x^2 cancels from the top and the bottom, leaving $x^3 = x^{5-2}$
7. (Challenge) Complete Venn diagram B for $6a^2b$ and $9ab^2$. Use it to simplify $\frac{6a^2b}{9ab^2}$. **The HCF is $3ab$, so $\frac{6a^2b}{9ab^2} = \frac{2a}{3b}$**

Worked Solution: Starter 1, Question 1

Question: Simplify $\sqrt{20}$.

Worked Solution: Starter 1, Question 1

Question: Simplify $\sqrt{20}$.

Solution: $20 = 4 \times 5$, and 4 is a square number.

So $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$.

Worked Solution: Starter 1, Question 2

Question: Work out 2^{-1} and $9^{\frac{1}{2}}$.

Worked Solution: Starter 1, Question 2

Question: Work out 2^{-1} and $9^{\frac{1}{2}}$.

Solution: A negative index means the reciprocal: $2^{-1} = \frac{1}{2}$.

A power of $\frac{1}{2}$ means the square root: $9^{\frac{1}{2}} = \sqrt{9} = 3$.

Worked Solution: Starter 1, Question 3

Question: Complete the factor tree. Write 84 as a product of prime factors in index form.

Worked Solution: Starter 1, Question 3

Question: Complete the factor tree. Write 84 as a product of prime factors in index form.

Solution: Split $84 = 2 \times 42$. Then $42 = 2 \times 21$, and $21 = 3 \times 7$.

So $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$.

In the tree, the missing box is 21, which splits into the primes 3 and 7.

Worked Solution: Starter 1, Question 4

Question: $60 = 2^2 \times 3 \times 5$. Complete Venn diagram A for 60 and 84, then find the HCF and the LCM of 60 and 84.

Worked Solution: Starter 1, Question 4

Question: $60 = 2^2 \times 3 \times 5$. Complete Venn diagram A for 60 and 84, then find the HCF and the LCM of 60 and 84.

Solution: Write $84 = 2^2 \times 3 \times 7$.

The shared prime factors are 2^2 and 3, so the HCF is $2^2 \times 3 = 12$.

The LCM uses every prime factor to its highest power:

$$2^2 \times 3 \times 5 \times 7 = 420.$$

In Venn diagram A, put 2, 2, 3 in the intersection, 5 only on the 60 side, and 7 only on the 84 side.

Worked Solution: Starter 1, Question 5

Question: Simplify $\frac{60}{84}$ by dividing by the HCF. Which numbers in Venn diagram A are not in the HCF?

Worked Solution: Starter 1, Question 5

Question: Simplify $\frac{60}{84}$ by dividing by the HCF. Which numbers in Venn diagram A are not in the HCF?

Solution: From Question 4, the HCF is 12.

$$\frac{60}{84} = \frac{60 \div 12}{84 \div 12} = \frac{5}{7}.$$

The numbers not in the HCF are 5 (only in 60) and 7 (only in 84); these are the numbers left in the simplified fraction.

Worked Solution: Starter 1, Question 6

Question: Simplify $\frac{x^5}{x^2}$ by writing out the xs and cancelling.

Worked Solution: Starter 1, Question 6

Question: Simplify $\frac{x^5}{x^2}$ by writing out the xs and cancelling.

Solution: $\frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x}$.

Cancel two xs from the top and bottom: $\frac{x \times x \times x}{1} = x^3$.

By index laws, this is $x^{5-2} = x^3$.

Worked Solution: Starter 1, Question 7

Question: (Challenge) Complete Venn diagram B for $6a^2b$ and $9ab^2$. Use it to simplify $\frac{6a^2b}{9ab^2}$.

Worked Solution: Starter 1, Question 7

Question: (Challenge) Complete Venn diagram B for $6a^2b$ and $9ab^2$. Use it to simplify $\frac{6a^2b}{9ab^2}$.

Solution: $6a^2b = 2 \times 3 \times a \times a \times b$.

$9ab^2 = 3 \times 3 \times a \times b \times b$.

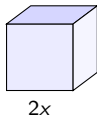
The shared factors are 3, a and b , so the HCF is $3ab$.

The remaining factors are $2a$ on the $6a^2b$ side and $3b$ on the $9ab^2$ side.

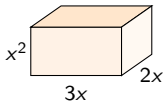
So $\frac{6a^2b}{9ab^2} = \frac{3ab \times 2a}{3ab \times 3b} = \frac{2a}{3b}$.

Starter 2: Index Laws

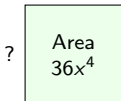
Cube



Cuboid



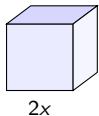
Square



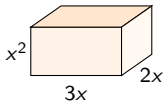
1. Write 72 as a product of prime factors in index form.
2. Simplify $\sqrt{8} + \sqrt{18}$.
3. Simplify $x^4 \times x^3$ and $y^8 \div y^2$.
4. Write 2^{-3} and x^{-2} as fractions.
5. Find the volume of the cube. Is it $2x^3$ or $8x^3$?
6. Find the volume of the cuboid.
7. The square has area $36x^4$, so its side length is $(36x^4)^{\frac{1}{2}}$. Simplify this.
8. (Extension) Complete the working.
$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{\square^2} = \square$$
9. (Challenge) Divide the volume of the cube by the volume of the cuboid. Write the answer as a fraction in its simplest form.

Starter 2: Index Laws

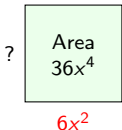
Cube



Cuboid



Square



- Write 72 as a product of prime factors in index form.
 $72 = 2^3 \times 3^2$
- Simplify $\sqrt{8} + \sqrt{18}$.
 $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$
- Simplify $x^4 \times x^3$ and $y^8 \div y^2$. x^7 and y^6
- Write 2^{-3} and x^{-2} as fractions. $\frac{1}{8}$ and $\frac{1}{x^2}$
- Find the volume of the cube. Is it $2x^3$ or $8x^3$?
 $(2x)^3 = 2^3 x^3 = 8x^3$
- Find the volume of the cuboid.
 $3x \times 2x \times x^2 = 6x^4$
- The square has area $36x^4$, so its side length is $(36x^4)^{\frac{1}{2}}$. Simplify this. $36^{\frac{1}{2}} \times x^{4 \times \frac{1}{2}} = 6x^2$
- (Extension) Complete the working.
$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$$
- (Challenge) Divide the volume of the cube by the volume of the cuboid. Write the answer as a fraction in its simplest form. $\frac{8x^3}{6x^4} = \frac{4}{3x}$

Worked Solution: Starter 2, Question 1

Question: Write 72 as a product of prime factors in index form.

Worked Solution: Starter 2, Question 1

Question: Write 72 as a product of prime factors in index form.

Solution: $72 = 8 \times 9$.

$8 = 2^3$ and $9 = 3^2$.

So $72 = 2^3 \times 3^2$.

Worked Solution: Starter 2, Question 2

Question: Simplify $\sqrt{8} + \sqrt{18}$.

Worked Solution: Starter 2, Question 2

Question: Simplify $\sqrt{8} + \sqrt{18}$.

Solution: $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}.$$

$$\text{So } \sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}.$$

Worked Solution: Starter 2, Question 3

Question: Simplify $x^4 \times x^3$ and $y^8 \div y^2$.

Worked Solution: Starter 2, Question 3

Question: Simplify $x^4 \times x^3$ and $y^8 \div y^2$.

Solution: For multiplication, add the indices: $x^4 \times x^3 = x^{4+3} = x^7$.
For division, subtract the indices: $y^8 \div y^2 = y^{8-2} = y^6$.

Worked Solution: Starter 2, Question 4

Question: Write 2^{-3} and x^{-2} as fractions.

Worked Solution: Starter 2, Question 4

Question: Write 2^{-3} and x^{-2} as fractions.

Solution: A negative index means take the reciprocal.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$$

$$x^{-2} = \frac{1}{x^2}.$$

Worked Solution: Starter 2, Question 5

Question: Find the volume of the cube. Is it $2x^3$ or $8x^3$?

Worked Solution: Starter 2, Question 5

Question: Find the volume of the cube. Is it $2x^3$ or $8x^3$?

Solution: The cube has side length $2x$.

$$\text{Volume} = (2x)^3 = 2^3 \times x^3 = 8x^3.$$

So it is $8x^3$, not $2x^3$; the 2 must also be cubed.

Worked Solution: Starter 2, Question 6

Question: Find the volume of the cuboid.

Worked Solution: Starter 2, Question 6

Question: Find the volume of the cuboid.

Solution: The cuboid has dimensions $3x$, x^2 and $2x$.

$$\text{Volume} = 3x \times x^2 \times 2x.$$

Multiply the numbers: $3 \times 2 = 6$.

Add the indices of x : $x^1 \times x^2 \times x^1 = x^{1+2+1} = x^4$.

So the volume is $6x^4$.

Worked Solution: Starter 2, Question 7

Question: The square has area $36x^4$, so its side length is $(36x^4)^{\frac{1}{2}}$. Simplify this.

Worked Solution: Starter 2, Question 7

Question: The square has area $36x^4$, so its side length is $(36x^4)^{\frac{1}{2}}$. Simplify this.

$$\text{Solution: } (36x^4)^{\frac{1}{2}} = 36^{\frac{1}{2}} \times (x^4)^{\frac{1}{2}}.$$

$$36^{\frac{1}{2}} = \sqrt{36} = 6.$$

$$(x^4)^{\frac{1}{2}} = x^{4 \times \frac{1}{2}} = x^2.$$

So the side length is $6x^2$.

Worked Solution: Starter 2, Question 8

Question: (Extension) Complete the working:

$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{\square^2} = \square.$$

Worked Solution: Starter 2, Question 8

Question: (Extension) Complete the working:

$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{\square^2} = \square.$$

Solution: $\sqrt[3]{8} = 2$ because $2^3 = 8$.

So $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$.

Therefore $8^{-\frac{2}{3}} = \frac{1}{4}$.

The blanks are 2 and $\frac{1}{4}$.

Worked Solution: Starter 2, Question 9

Question: (Challenge) Divide the volume of the cube by the volume of the cuboid. Write the answer as a fraction in its simplest form.

Worked Solution: Starter 2, Question 9

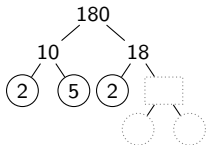
Question: (Challenge) Divide the volume of the cube by the volume of the cuboid. Write the answer as a fraction in its simplest form.

Solution: From Questions 5 and 6, the cube has volume $8x^3$ and the cuboid has volume $6x^4$.

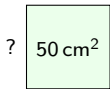
So the quotient is $\frac{8x^3}{6x^4} = \frac{8}{6} \times \frac{x^3}{x^4} = \frac{4}{3} \times \frac{1}{x} = \frac{4}{3x}$.

Starter 3: Surds & Single Brackets

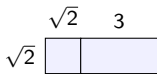
Factor tree



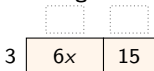
Square



Rectangle A



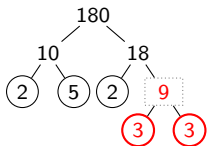
Rectangle B



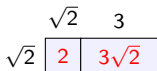
- Find the HCF of 18 and 24.
- Work out $16^{\frac{1}{2}}$ and $16^{-\frac{1}{2}}$.
- Complete the factor tree. Write 180 as a product of prime factors in index form.
- Use the prime factors to simplify $\sqrt{180}$.
- The square has area 50 cm^2 . Find its side length as a simplified surd, and its perimeter.
- Complete rectangle A to expand $\sqrt{2}(\sqrt{2} + 3)$.
- Complete rectangle B to factorise $6x + 15$. Then simplify $\frac{6x + 15}{3}$.
- (Challenge) Sam says $\frac{6 + \sqrt{18}}{3} = 2 + \sqrt{18}$. Explain Sam's mistake, then simplify it correctly.

Starter 3: Surds & Single Brackets

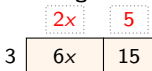
Factor tree



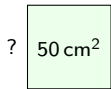
Rectangle A



Rectangle B



Square



$5\sqrt{2} \text{ cm}$

- Find the HCF of 18 and 24. **6**
- Work out $16^{\frac{1}{2}}$ and $16^{-\frac{1}{2}}$. **4 and $\frac{1}{4}$**
- Complete the factor tree. Write 180 as a product of prime factors in index form.
 $180 = 2^2 \times 3^2 \times 5$
- Use the prime factors to simplify $\sqrt{180}$.
 $\sqrt{2^2 \times 3^2 \times 5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}$
- The square has area 50 cm^2 . Find its side length as a simplified surd, and its perimeter.
 $\sqrt{50} = 5\sqrt{2} \text{ cm}$; $4 \times 5\sqrt{2} = 20\sqrt{2} \text{ cm}$
- Complete rectangle A to expand $\sqrt{2}(\sqrt{2} + 3)$.
 $2 + 3\sqrt{2}$
- Complete rectangle B to factorise $6x + 15$. Then simplify $\frac{6x + 15}{3}$. **$3(2x + 5)$; $2x + 5$, not $2x + 15$**
- (Challenge) Sam says $\frac{6 + \sqrt{18}}{3} = 2 + \sqrt{18}$. Explain Sam's mistake, then simplify it correctly.
Divide both terms by 3: $2 + \frac{3\sqrt{2}}{3} = 2 + \sqrt{2}$

Worked Solution: Starter 3, Question 1

Question: Find the HCF of 18 and 24.

Worked Solution: Starter 3, Question 1

Question: Find the HCF of 18 and 24.

Solution: $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$.

The shared prime factors are 2 and 3.

So the HCF is $2 \times 3 = 6$.

Worked Solution: Starter 3, Question 2

Question: Work out $16^{\frac{1}{2}}$ and $16^{-\frac{1}{2}}$.

Worked Solution: Starter 3, Question 2

Question: Work out $16^{\frac{1}{2}}$ and $16^{-\frac{1}{2}}$.

Solution: $16^{\frac{1}{2}} = \sqrt{16} = 4$.

$$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{4}.$$

Worked Solution: Starter 3, Question 3

Question: Complete the factor tree. Write 180 as a product of prime factors in index form.

Worked Solution: Starter 3, Question 3

Question: Complete the factor tree. Write 180 as a product of prime factors in index form.

Solution: $180 = 10 \times 18$.

$10 = 2 \times 5$ and $18 = 2 \times 9 = 2 \times 3 \times 3$.

So $180 = 2 \times 5 \times 2 \times 3 \times 3 = 2^2 \times 3^2 \times 5$.

In the tree, the missing box is 9, which splits into 3 and 3.

Worked Solution: Starter 3, Question 4

Question: Use the prime factors to simplify $\sqrt{180}$.

Worked Solution: Starter 3, Question 4

Question: Use the prime factors to simplify $\sqrt{180}$.

Solution: From Question 3, $180 = 2^2 \times 3^2 \times 5$.

So $\sqrt{180} = \sqrt{2^2 \times 3^2 \times 5} = \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{5} = 2 \times 3 \times \sqrt{5} = 6\sqrt{5}$.

Worked Solution: Starter 3, Question 5

Question: The square has area 50 cm^2 . Find its side length as a simplified surd, and its perimeter.

Worked Solution: Starter 3, Question 5

Question: The square has area 50 cm^2 . Find its side length as a simplified surd, and its perimeter.

Solution: Side length $= \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \text{ cm}$.

Perimeter $= 4 \times 5\sqrt{2} = 20\sqrt{2} \text{ cm}$.

Worked Solution: Starter 3, Question 6

Question: Complete rectangle A to expand $\sqrt{2}(\sqrt{2} + 3)$.

Worked Solution: Starter 3, Question 6

Question: Complete rectangle A to expand $\sqrt{2}(\sqrt{2} + 3)$.

Solution: $\sqrt{2}(\sqrt{2} + 3) = \sqrt{2} \times \sqrt{2} + \sqrt{2} \times 3$.

$$\sqrt{2} \times \sqrt{2} = 2.$$

$$\sqrt{2} \times 3 = 3\sqrt{2}.$$

So the expansion is $2 + 3\sqrt{2}$.

The two rectangle areas are 2 and $3\sqrt{2}$.

Worked Solution: Starter 3, Question 7

Question: Complete rectangle B to factorise $6x + 15$. Then simplify $\frac{6x + 15}{3}$.

Worked Solution: Starter 3, Question 7

Question: Complete rectangle B to factorise $6x + 15$. Then simplify $\frac{6x + 15}{3}$.

Solution: The HCF of $6x$ and 15 is 3 .

So $6x + 15 = 3(2x + 5)$.

Then $\frac{6x+15}{3} = \frac{3(2x+5)}{3} = 2x + 5$.

It is not $2x + 15$; both terms must be divided by 3 .

Worked Solution: Starter 3, Question 8

Question: (Challenge) Sam says $\frac{6 + \sqrt{18}}{3} = 2 + \sqrt{18}$. Explain Sam's mistake, then simplify it correctly.

Worked Solution: Starter 3, Question 8

Question: (Challenge) Sam says $\frac{6 + \sqrt{18}}{3} = 2 + \sqrt{18}$. Explain Sam's mistake, then simplify it correctly.

Solution: Sam divided only the 6 by 3, but both terms in the numerator must be divided by 3.

Correctly: $\frac{6 + \sqrt{18}}{3} = \frac{6}{3} + \frac{\sqrt{18}}{3} = 2 + \frac{3\sqrt{2}}{3} = 2 + \sqrt{2}$.

Starter 4: Double Brackets

Grid A

	x	5
x	x^2	
2		

Grid B

	x	
x	x^2	
		12

Area = $x^2 + 7x + 12$

1. Is 3^{-2} equal to -9 , -6 or $\frac{1}{9}$?
2. Simplify $\sqrt{50} - \sqrt{8}$. Is it $\sqrt{42}$?
3. Complete grid A to expand $(x + 2)(x + 5)$.
4. Write 12 as a product of prime factors, then list its factor pairs. Which pair adds to 7?
5. Complete grid B to factorise $x^2 + 7x + 12$.
6. A rectangle has area $x^2 + 7x + 12$ and width $x + 3$. Find its length. Use this to simplify $\frac{x^2 + 7x + 12}{x + 3}$.
7. Factorise $x^2 - 9$. Use the difference of two squares to expand $(3 + \sqrt{2})(3 - \sqrt{2})$.
8. (Challenge) $391 = 20^2 - 3^2$. Use this to write 391 as a product of prime factors.

Starter 4: Double Brackets

Grid A

	x	5
x	x^2	$5x$
2	$2x$	10

Grid B

	x	4
x	x^2	$4x$
3	$3x$	12

Area = $x^2 + 7x + 12$

- Is 3^{-2} equal to -9 , -6 or $\frac{1}{9}$? $\frac{1}{3^2} = \frac{1}{9}$
- Simplify $\sqrt{50} - \sqrt{8}$. Is it $\sqrt{42}$? **No:**
 $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$
- Complete grid A to expand $(x + 2)(x + 5)$.
 $x^2 + 7x + 10$
- Write 12 as a product of prime factors, then list its factor pairs. Which pair adds to 7? $2^2 \times 3$;
 $1 \times 12, 2 \times 6, 3 \times 4; 3 + 4 = 7$
- Complete grid B to factorise $x^2 + 7x + 12$.
 $(x + 3)(x + 4)$, in either order
- A rectangle has area $x^2 + 7x + 12$ and width $x + 3$. Find its length. Use this to simplify $\frac{x^2 + 7x + 12}{x + 3}$.
 $\frac{(x + 3)(x + 4)}{x + 3} = x + 4$
- Factorise $x^2 - 9$. Use the difference of two squares to expand $(3 + \sqrt{2})(3 - \sqrt{2})$. $(x + 3)(x - 3)$;
 $9 - 2 = 7$
- (Challenge) $391 = 20^2 - 3^2$. Use this to write 391 as a product of prime factors.
 $(20 + 3)(20 - 3) = 23 \times 17$, both prime

Worked Solution: Starter 4, Question 1

Question: Is 3^{-2} equal to -9 , -6 or $\frac{1}{9}$?

Worked Solution: Starter 4, Question 1

Question: Is 3^{-2} equal to -9 , -6 or $\frac{1}{9}$?

Solution: A negative index means the reciprocal, not a negative number.

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$$

So it is $\frac{1}{9}$, not -9 or -6 .

Worked Solution: Starter 4, Question 2

Question: Simplify $\sqrt{50} - \sqrt{8}$. Is it $\sqrt{42}$?

Worked Solution: Starter 4, Question 2

Question: Simplify $\sqrt{50} - \sqrt{8}$. Is it $\sqrt{42}$?

Solution: $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$.

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}.$$

So $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$.

It is not $\sqrt{42}$; you cannot subtract the numbers under the roots like that.

Worked Solution: Starter 4, Question 3

Question: Complete grid A to expand $(x + 2)(x + 5)$.

Worked Solution: Starter 4, Question 3

Question: Complete grid A to expand $(x + 2)(x + 5)$.

Solution: Expand by multiplying each term in the first bracket by each term in the second:

$$x \times x = x^2, x \times 5 = 5x, 2 \times x = 2x, 2 \times 5 = 10.$$

$$\text{Sum: } x^2 + 5x + 2x + 10 = x^2 + 7x + 10.$$

The four grid areas are x^2 , $5x$, $2x$ and 10 .

Worked Solution: Starter 4, Question 4

Question: Write 12 as a product of prime factors, then list its factor pairs. Which pair adds to 7?

Worked Solution: Starter 4, Question 4

Question: Write 12 as a product of prime factors, then list its factor pairs. Which pair adds to 7?

Solution: $12 = 2^2 \times 3$.

Factor pairs: 1×12 , 2×6 , 3×4 .

Their sums are 13, 8 and 7.

So the pair 3 and 4 adds to 7.

Worked Solution: Starter 4, Question 5

Question: Complete grid B to factorise $x^2 + 7x + 12$.

Worked Solution: Starter 4, Question 5

Question: Complete grid B to factorise $x^2 + 7x + 12$.

Solution: We need two numbers that multiply to 12 and add to 7.

From Question 4, the numbers are 3 and 4.

So $x^2 + 7x + 12 = (x + 3)(x + 4)$.

The grid has missing lengths 4 and 3, and areas $4x$ and $3x$.

Worked Solution: Starter 4, Question 6

Question: A rectangle has area $x^2 + 7x + 12$ and width $x + 3$. Find its length. Use this to simplify $\frac{x^2 + 7x + 12}{x + 3}$.

Worked Solution: Starter 4, Question 6

Question: A rectangle has area $x^2 + 7x + 12$ and width $x + 3$. Find its length. Use this to simplify $\frac{x^2 + 7x + 12}{x + 3}$.

Solution: Factorise the area: $x^2 + 7x + 12 = (x + 3)(x + 4)$.

Since the width is $x + 3$, the length is $x + 4$.

So $\frac{x^2 + 7x + 12}{x + 3} = \frac{(x + 3)(x + 4)}{x + 3} = x + 4$, provided $x \neq -3$.

Worked Solution: Starter 4, Question 7

Question: Factorise $x^2 - 9$. Use the difference of two squares to expand $(3 + \sqrt{2})(3 - \sqrt{2})$.

Worked Solution: Starter 4, Question 7

Question: Factorise $x^2 - 9$. Use the difference of two squares to expand $(3 + \sqrt{2})(3 - \sqrt{2})$.

Solution: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$.

For the surds, use $(a + b)(a - b) = a^2 - b^2$:

$$(3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2 = 9 - 2 = 7.$$

Worked Solution: Starter 4, Question 8

Question: (Challenge) $391 = 20^2 - 3^2$. Use this to write 391 as a product of prime factors.

Worked Solution: Starter 4, Question 8

Question: (Challenge) $391 = 20^2 - 3^2$. Use this to write 391 as a product of prime factors.

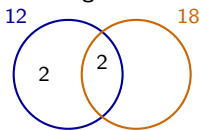
Solution: Use the difference of two squares:

$$391 = 20^2 - 3^2 = (20 + 3)(20 - 3) = 23 \times 17.$$

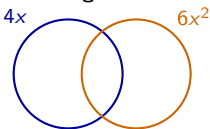
Both 23 and 17 are prime, so $391 = 23 \times 17$.

Starter 5: LCM & Fractions

Venn diagram A



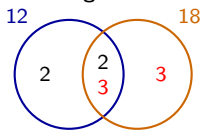
Venn diagram B



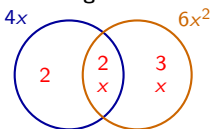
1. Simplify $a^3 \times a^{-1}$.
2. Work out $\sqrt{8} \times \sqrt{2}$.
3. $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. Complete Venn diagram A, then find the HCF and the LCM of 12 and 18.
4. Complete: $\frac{5}{12} + \frac{7}{18} = \frac{\boxed{}}{36} + \frac{\boxed{}}{36} = \frac{\boxed{}}{36}$
5. Write $\frac{x}{12} - \frac{x}{18}$ as a single fraction.
6. Work out $\frac{3}{4} \times \frac{8}{9}$. Then simplify $\frac{3x}{4} \div \frac{9x}{10}$.
7. $4x = 2 \times 2 \times x$ and $6x^2 = 2 \times 3 \times x \times x$. Complete Venn diagram B, then find the LCM of $4x$ and $6x^2$.
8. (Challenge) Write $\frac{5}{4x} + \frac{1}{6x^2}$ as a single fraction.

Starter 5: LCM & Fractions

Venn diagram A



Venn diagram B



1. Simplify $a^3 \times a^{-1}$.
 $a^{3+(-1)} = a^2$
2. Work out $\sqrt{8} \times \sqrt{2}$.
 $\sqrt{16} = 4$
3. $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. Complete Venn diagram A, then find the HCF and the LCM of 12 and 18. **HCF = 6; LCM = $2^2 \times 3^2 = 36$**
4. Complete: $\frac{5}{12} + \frac{7}{18} = \frac{15}{36} + \frac{14}{36} = \frac{29}{36}$
5. Write $\frac{x}{12} - \frac{x}{18}$ as a single fraction.
 $\frac{3x}{36} - \frac{2x}{36} = \frac{x}{36}$
6. Work out $\frac{3}{4} \times \frac{8}{9}$. Then simplify $\frac{3x}{4} \div \frac{9x}{10}$.
 $\frac{24}{36} = \frac{2}{3}$; $\frac{3x}{4} \times \frac{10}{9x} = \frac{30x}{36x} = \frac{5}{6}$
7. $4x = 2 \times 2 \times x$ and $6x^2 = 2 \times 3 \times x \times x$. Complete Venn diagram B, then find the LCM of $4x$ and $6x^2$. **$2 \times 2 \times 3 \times x \times x = 12x^2$**
8. (Challenge) Write $\frac{5}{4x} + \frac{1}{6x^2}$ as a single fraction.
 $\frac{15x}{12x^2} + \frac{2}{12x^2} = \frac{15x+2}{12x^2}$

Worked Solution: Starter 5, Question 1

Question: Simplify $a^3 \times a^{-1}$.

Worked Solution: Starter 5, Question 1

Question: Simplify $a^3 \times a^{-1}$.

Solution: When multiplying powers with the same base, add the indices:

$$a^3 \times a^{-1} = a^{3+(-1)} = a^2.$$

Worked Solution: Starter 5, Question 2

Question: Work out $\sqrt{8} \times \sqrt{2}$.

Worked Solution: Starter 5, Question 2

Question: Work out $\sqrt{8} \times \sqrt{2}$.

Solution: $\sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = 4$.

Worked Solution: Starter 5, Question 3

Question: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. Complete Venn diagram A, then find the HCF and the LCM of 12 and 18.

Worked Solution: Starter 5, Question 3

Question: $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. Complete Venn diagram A, then find the HCF and the LCM of 12 and 18.

Solution: Write $12 = 2 \times 2 \times 3$ and $18 = 2 \times 3 \times 3$.

The shared factors are 2 and 3, so the HCF is $2 \times 3 = 6$.

The LCM uses every prime factor to its highest power:

$$2^2 \times 3^2 = 4 \times 9 = 36.$$

In the Venn diagram, put 2 and 3 in the intersection, 2 only on the 12 side, and 3 only on the 18 side.

Worked Solution: Starter 5, Question 4

Question: Complete: $\frac{5}{12} + \frac{7}{18} = \frac{\square}{36} + \frac{\square}{36} = \frac{\square}{36}$.

Worked Solution: Starter 5, Question 4

Question: Complete: $\frac{5}{12} + \frac{7}{18} = \frac{\square}{36} + \frac{\square}{36} = \frac{\square}{36}$.

Solution: The LCM of 12 and 18 is 36.

$$\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}.$$

$$\frac{7}{18} = \frac{7 \times 2}{18 \times 2} = \frac{14}{36}.$$

$$\text{So } \frac{5}{12} + \frac{7}{18} = \frac{15}{36} + \frac{14}{36} = \frac{29}{36}.$$

The blanks are 15, 14 and 29.

Worked Solution: Starter 5, Question 5

Question: Write $\frac{x}{12} - \frac{x}{18}$ as a single fraction.

Worked Solution: Starter 5, Question 5

Question: Write $\frac{x}{12} - \frac{x}{18}$ as a single fraction.

Solution: The LCM of 12 and 18 is 36.

$$\frac{x}{12} = \frac{3x}{36} \text{ and } \frac{x}{18} = \frac{2x}{36}.$$

$$\text{So } \frac{x}{12} - \frac{x}{18} = \frac{3x}{36} - \frac{2x}{36} = \frac{x}{36}.$$

Worked Solution: Starter 5, Question 6

Question: Work out $\frac{3}{4} \times \frac{8}{9}$. Then simplify $\frac{3x}{4} \div \frac{9x}{10}$.

Worked Solution: Starter 5, Question 6

Question: Work out $\frac{3}{4} \times \frac{8}{9}$. Then simplify $\frac{3x}{4} \div \frac{9x}{10}$.

Solution: $\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9} = \frac{24}{36} = \frac{2}{3}$.

For the division, multiply by the reciprocal:

$$\frac{3x}{4} \div \frac{9x}{10} = \frac{3x}{4} \times \frac{10}{9x} = \frac{30x}{36x} = \frac{30}{36} = \frac{5}{6}, \text{ assuming } x \neq 0.$$

Worked Solution: Starter 5, Question 7

Question: $4x = 2 \times 2 \times x$ and $6x^2 = 2 \times 3 \times x \times x$. Complete Venn diagram B, then find the LCM of $4x$ and $6x^2$.

Worked Solution: Starter 5, Question 7

Question: $4x = 2 \times 2 \times x$ and $6x^2 = 2 \times 3 \times x \times x$. Complete Venn diagram B, then find the LCM of $4x$ and $6x^2$.

Solution: $4x = 2^2 \times x$ and $6x^2 = 2 \times 3 \times x^2$.

The LCM uses every factor to its highest power:

$$2^2 \times 3 \times x^2 = 4 \times 3 \times x^2 = 12x^2.$$

In the Venn diagram, put 2 and x in the intersection, 2 only on the $4x$ side, and 3 and x only on the $6x^2$ side.

Worked Solution: Starter 5, Question 8

Question: (Challenge) Write $\frac{5}{4x} + \frac{1}{6x^2}$ as a single fraction.

Worked Solution: Starter 5, Question 8

Question: (Challenge) Write $\frac{5}{4x} + \frac{1}{6x^2}$ as a single fraction.

Solution: The LCM of $4x$ and $6x^2$ is $12x^2$.

$$\frac{5}{4x} = \frac{5 \times 3x}{4x \times 3x} = \frac{15x}{12x^2}.$$

$$\frac{1}{6x^2} = \frac{1 \times 2}{6x^2 \times 2} = \frac{2}{12x^2}.$$

$$\text{So } \frac{5}{4x} + \frac{1}{6x^2} = \frac{15x}{12x^2} + \frac{2}{12x^2} = \frac{15x+2}{12x^2}.$$