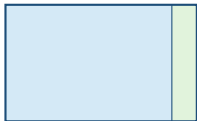


Lattice Multiplication

	4	7	
1	1	2	3
6	2	4	6
	9	2	

Pole
1. Area



Multiplying is finding the **area** of a **pole**
prostokąt
rectangle .

Siatka
2. Grid

×	20	3
10	200	30
4	80	12

prostokąt
The **rectangle** written as a table.

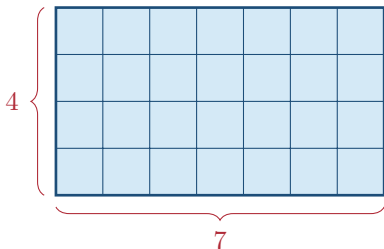
Every zero shown.

Krata
3. Lattice



iloczynny
The same **products** , with the
przekątne
diagonals doing the
wartość pozycyjna
place value .

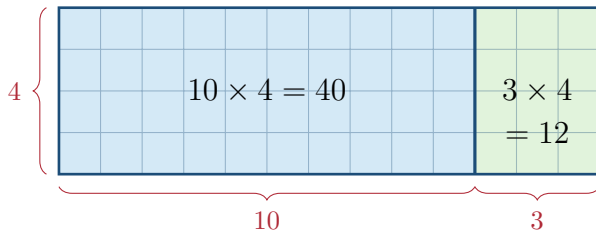
Mnożenie pole Multiplication area



- A **prostokąt** **rectangle** 7 wide and 4 tall.
- Count the **kwadraty** **squares** : $7 \times 4 = 28$.

- Every **mnożenie** **multiplication** question can be represented as a **prostokąt** **rectangle**.

prostokąt
rectangle

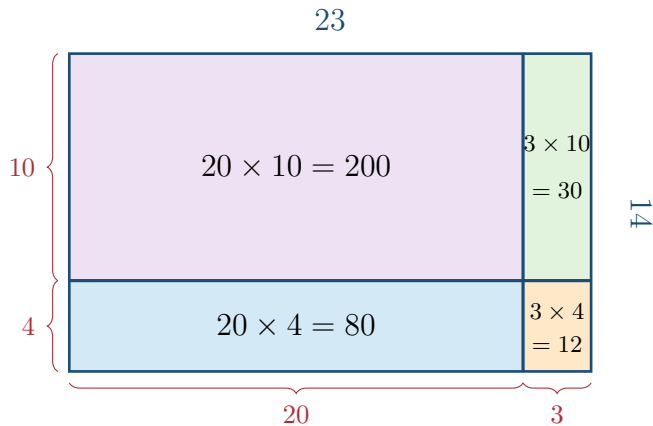


$$13 \times 4 = (10 \times 4) + (3 \times 4) = 40 + 12 = 52$$

We split 13 into the parts we can actually multiply in our heads: its dziesiątki *tens* and its jedności *units* .

cyfry
digits

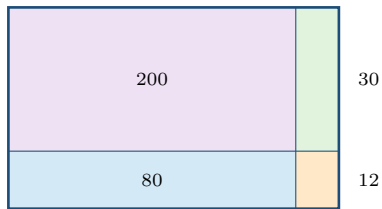
cyfry
digits



$$23 \times 14 = 200 + 30 + 80 + 12 = 322$$

prostokąt
rectangle

siatka
grid



×	20	3
10	200	30
4	80	12

$$200 + 30 + 80 + 12 = 322$$

siatka

- The **grid** is like the picture, but not drawn to scale.

$\times$	40	7
30	1200	210
6	240	42

$$4 \times 3 = 12 \quad \text{so } 40 \times 30 = 1200$$

$$4 \times 6 = 24 \quad \text{so } 40 \times 6 = 240$$

$$7 \times 3 = 21 \quad \text{so } 7 \times 30 = 210$$

$$7 \times 6 = 42 \quad \text{so } 7 \times 6 = 42$$

$$1200 + 240 + 210 + 42 = 1692, \quad \text{so } 47 \times 36 = 1692$$

(a) $34 \times 12 =$

(b) $26 \times 45 =$

(c) $53 \times 27 =$

(d) $18 \times 63 =$

(e) $72 \times 84 =$

(f) $95 \times 48 =$

×		

Challenge. 124×36 . How many boxes does

your **siatka**
grid need now, and why?

Answer:

siatka grid

(a) $34 \times 12 = 408$

(b) $26 \times 45 = 1170$

(c) $53 \times 27 = 1431$

(d) $18 \times 63 = 1134$

(e) $72 \times 84 = 6048$

(f) $95 \times 48 = 4560$

×		

Challenge. 124×36 . How many boxes does

your **siatka**
grid need now, and why?

Answer: $3 \times 2 = 6$ boxes, one for each pair of
wartości pozycyjne
place values ; $124 \times 36 = 4464$

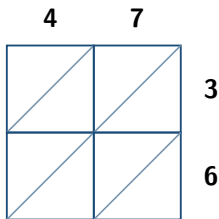
siatka grid

×	300	40	7
600	180000	24000	4200
50	15000	2000	350
8	2400	320	56

- The method is fine for bigger numbers...
- But it is a lot of writing long numbers

- The **lattice** keeps the same nine **products** but writes only the **digits** — and uses **przekątne** *diagonals* instead of zeros to keep track of **wartość pozycyjna** *place value* .

krata lattice



- 47×36 .
- One *column* for each *cyfra* *digit* of 47, written across the top.
- One *row* for each *cyfra* *digit* of 36, written down the right.
- Draw a *przekątna* *diagonal* in every cell, running from top-right down to bottom-left.

tabliczka mnożenia times-table

	4	7	
3	12	21	
6	24	42	

cyfry

- Multiply each of the combinations of digits in the relevant cell.

Dziesiątki

trójkąt

- Tens** go in the upper-left triangle, **units** in the lower-right.

$$4 \times 3 = 12 \rightarrow \boxed{1} / \boxed{2}$$

$$7 \times 3 = 21 \rightarrow \boxed{2} / \boxed{1}$$

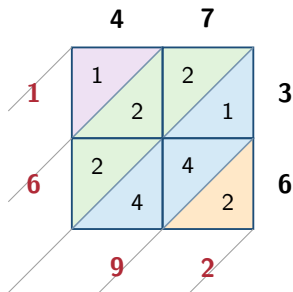
$$4 \times 6 = 24 \rightarrow \boxed{2} / \boxed{4}$$

$$7 \times 6 = 42 \rightarrow \boxed{4} / \boxed{2}$$

A one-digit answer such as $3 \times 2 = 6$ is written 0/6.

dodawaj
add

przekątne
diagonals

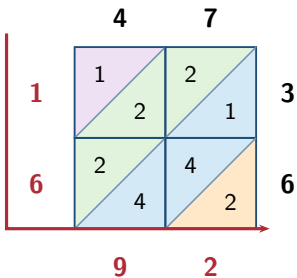


Start at the bottom right and work up the strips.

jedności units	2	= 2
dziesiątki tens	4 + 4 + 1	= 9
setki hundreds	2 + 2 + 2	= 6
tysiące thousands	1	= 1

- Each shaded strip is one wartość pozycyjna
place-value column.

- Write each total just outside the krata
lattice



- Read **down the left-hand side**, then **along the bottom**.
- 1, 6, 9, 2

- $47 \times 36 = 1692$

siatka

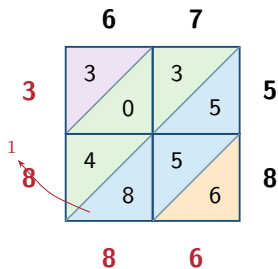
Compare with the **grid** method:

$1200 + 240 + 210 + 42 = 1692$. its the same four

iloczynny

products, the same answer — but no long addition.

przekątna
diagonal



$$67 \times 58$$

jedności

units

6

= 6

dziesiątki

tens

5 + 8 + 5

= 18

przenosić

write 8,

carry

1

setki

hundreds

4 + 3 + 0 + 1

= 8

tysiące

thousands

3

= 3

- If a strip totals 10 or more, write down the

jedności cyfra

przenosić

dziesiątki

units

digit

and

carry

the

tens

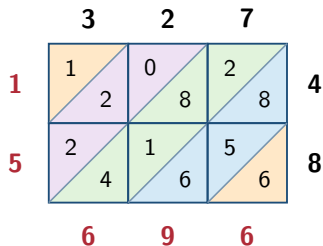
into the next strip up-left.

przekątne diagonals

	4	7	
Th	1 2	2 1	3
H	2 4	4 2	6
	T	U	

- The top-left cell really means $40 \times 30 = 1200$,
 setki
 i.e. 12 hundreds.
- Its 2 sits on the hundreds diagonal and its 1
 przekątna
 sits one diagonal further left — the
 tysiące
 thousands. Exactly right.
- Every step you move left or up in the krata
 lattice
 multiplies by 10; both moves shift you one
 przekątna
 diagonal along.

krata lattice



jedności

units

6

= 6

dziesiątki

tens

5 + 6 + 8

= 19 → 9,

setki

hundreds

1 + 4 + 2 + 8 + 1

= 16 → 6,

tyśiące

thousands

2 + 0 + 2 + 1

= 5

dziesiątki tysięcy

ten thous.

1

= 1

cyfry

cyfry

- Three digits by two digits $\Rightarrow$ a 3×2

krata

lattice.

- Nothing else changes.

krata
lattice

(a) $38 \times 24 =$

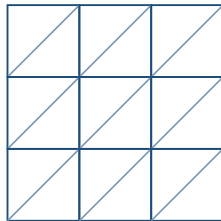
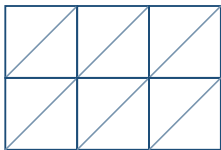
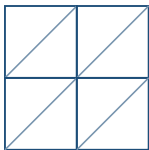
(b) $65 \times 43 =$

(c) $57 \times 89 =$

(d) $127 \times 36 =$

(e) $408 \times 57 =$

(f) $234 \times 156 =$



krata
lattice

(a) $38 \times 24 = 912$

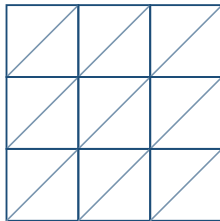
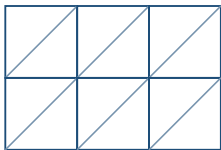
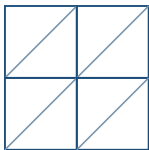
(b) $65 \times 43 = 2795$

(c) $57 \times 89 = 5073$

(d) $127 \times 36 = 4572$

(e) $408 \times 57 = 23\,256$

(f) $234 \times 156 = 36\,504$



And now the przecinek dziesiętny decimal point

szacować
estimate

$$4.7 \times 3.6$$

$$4.7 \approx 5 \quad 3.6 \approx 4 \quad \Rightarrow \quad \text{the answer is about } 20$$

1.692

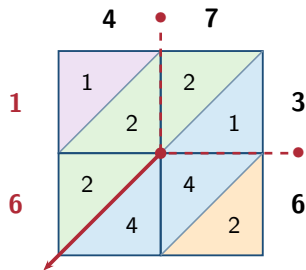
16.92

169.2

✓ the only one near 20

- The **krata** **lattice** will give the **cyfry** **digits** 1 6 9 2 whatever the **przecinki dziesiętne** **decimal points** are doing.
- We just need to work out where the **przecinek dziesiętny** **decimal point** goes.

przecinek dziesiętny decimal point



przecinki dziesiętne

1. Ignore the decimal points and build the lattice from the digits: 47 and 36.

przecinek dziesiętny

2. Mark the decimal point of the top number on the **top edge**, and of the side number on the **right edge**.

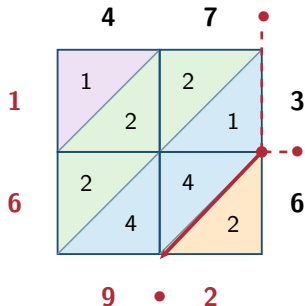
3. Slide one *down* and the other *across* until they *meet*.

4. From that meeting point, follow the przekatna

krata
lattice

przecinek dziesiętny
decimal point

$$47 \times 3.6$$



- 47 has no **przecinek dziesiętny** **decimal point** written, so it sits at the **far right** of the top edge.
- 3.6 puts its point between the two rows, as before.
- They meet lower down, so the **przekątna** **diagonal** **krata** **lattice** leaves the **lattice** one place further right.

$$47 \times 3.6 = 169.2$$

miejsca dziesiętne decimal places

$$4.7 \times 3.6 \quad 1 + 1 = 2 \text{ places} \Rightarrow 16.92$$

$$47 \times 3.6 \quad 0 + 1 = 1 \text{ place} \Rightarrow 169.2$$

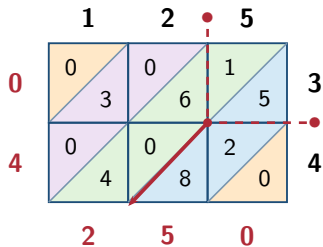
$$0.47 \times 3.6 \quad 2 + 1 = 3 \text{ places} \Rightarrow 1.692$$

$$0.47 \times 0.36 \quad 2 + 2 = 4 \text{ places} \Rightarrow 0.1692$$

- The **cyfry** **digits** 1692 come from the same **krata** **lattice** every single time.
- Total **miejsca dziesiętne** **decimal places** in the question = **miejsca dziesiętne** **decimal places** in the answer.
- The **przekątna** **diagonal** rule and the counting rule always agree — use whichever you prefer, and **oszacowanie** *check against your estimate* .

zero wiodące

Careful with (d): after counting four places you need a **leading zero** to fill the gap.



jedności

units

0

= 0

dziesiątki

tens

2 + 8 + 5

= 15 → 5,

setki

hundreds

0 + 4 + 1 + 6 + 1

= 12 → 2,

tysiące

thousands

0 + 0 + 3 + 1

= 4

dziesiątki tysięcy

ten thous.

0

= 0

Cyfry

- Digits: $125 \times 34 = 4250$.

- Points meet after the 2 on top and

przekątna

between the rows, so the diagonal exits

ułamki dziesiętne
decimals

(a) $2.4 \times 3.7 =$

(b) $6.3 \times 0.8 =$

(c) $5.2 \times 4.6 =$

(d) $0.46 \times 2.3 =$

(e) $13.4 \times 2.5 =$

(f) $0.85 \times 0.64 =$

oszacowanie

For each one, write your **estimate** before you start.

cyfry

Challenge 1. 1.25×0.48 . Why does the answer need fewer **digits** than you expect?

krata

cyfry

Challenge 2. A **lattice** for $\square.\square \times 4.5$ gives the **digits** 2790 and the answer 27.90. What was the first number?

przekątna

Challenge 3. Explain to your partner why a **diagonal** is the same thing as a **wartość pozycyjna**

ułamki dziesiętne
decimals

(a) $2.4 \times 3.7 = 8.88$

(b) $6.3 \times 0.8 = 5.04$

(c) $5.2 \times 4.6 = 23.92$

(d) $0.46 \times 2.3 = 1.058$

(e) $13.4 \times 2.5 = 33.5$

(f) $0.85 \times 0.64 = 0.544$

oszacowanie

For each one, write your **estimate** before you start.

cyfry

Challenge 1. 1.25×0.48 . Why does the answer need fewer **digits** than you expect?

krata

zera końcowe

$= 0.6$; the **lattice** gives 6000 and the **trailing zeros** disappear.

krata

cyfry

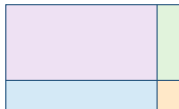
Challenge 2. A **lattice** for $\square.\square \times 4.5$ gives the **digits** 2790 and the answer 27.90. What

was the first number? **6.2**

przekątna

Challenge 3. Explain to your partner why a **diagonal** is the same thing as a **wartość pozycyjna**

Pole Area



Split each number into

wartości pozycyjne

place values ; the four

pola iloczynów

areas are the four products .

Siatka Grid

×	40	7
30	1200	210
6	240	42

Every zero written out. Reliable, but a

long addition at the end.

Krata Lattice

1	2
2	1
2	4
4	2

Cyfry przekątne

Digits only. The diagonals

przenoszą

carry the

wartość pozycyjna

place value . Read down,

then across.

Dziesiątki trójkąt jedności

- Tens in the upper-left triangle, units in the lower-right.

Dodawaj przekątne przenosząc

- Add down the diagonals from the bottom right. carrying up-left.