

Index Rules: Quick Questions

Mini whiteboard questions, with solutions

Contents

- Index rules with positive indices
- Index rules with negative indices
- Simplifying expressions with index laws
- Estimating roots and powers
- Indices of the form $\frac{1}{a}$
- Indices of the form $\frac{a}{b}$
- Simplifying expressions with negative fractional indices

Index rules with positive indices

True or false?

4^3 means $4 \times 4 \times 4$

True or false?

4^3 means $4 \times 4 \times 4$

True. The index counts how many 4s are multiplied together,
so $4^3 = 64$.

True or false?

$$2^5 \times 2^3 = 2^8$$

True or false?

$$2^5 \times 2^3 = 2^8$$

True. Five 2s multiplied by three 2s gives eight 2s, so the indices are added.

True or false?

$$2^5 \times 2^3 = 4^8$$

True or false?

$$2^5 \times 2^3 = 4^8$$

False. Only the indices are added; the base stays as 2.

The answer is $2^8 = 256$, not $4^8 = 65\,536$.

True or false?

$$\frac{7^8}{7^2} = 7^4$$

True or false?

$$\frac{7^8}{7^2} = 7^4$$

False. The indices are subtracted, not divided.

$$7^8 \div 7^2 = 7^{8-2} = 7^6$$

True or false?

$$(5^2)^3 = 5^5$$

True or false?

$$(5^2)^3 = 5^5$$

False. $(5^2)^3 = 5^2 \times 5^2 \times 5^2$, which is six 5s.

The indices are multiplied: 5^6 .

Simplify $2^3 \times 2^4$

Simplify $2^3 \times 2^4$

$(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ — seven 2s

$$2^3 \times 2^4 = 2^{3+4} = 2^7$$

Simplify $5^6 \div 5^2$

Simplify $5^6 \div 5^2$

$$5^{6-2} = 5^4$$

Simplify $7^5 \times 7$

Simplify $7^5 \times 7$

7 on its own is 7^1 .

$$7^{5+1} = 7^6$$

Simplify $(3^2)^4$

Simplify $(3^2)^4$

$$3^{2 \times 4} = 3^8$$

Simplify $4^3 \times 4^2 \times 4^5$

Simplify $4^3 \times 4^2 \times 4^5$

$$4^{3+2+5} = 4^{10}$$

Simplify $\frac{6^8}{6^3 \times 6^2}$

Simplify $\frac{6^8}{6^3 \times 6^2}$

$$6^3 \times 6^2 = 6^5$$

$$\frac{6^8}{6^5} = 6^{8-5} = 6^3$$

Simplify $(2^3)^2 \times 2^4$

Simplify $(2^3)^2 \times 2^4$

$$(2^3)^2 = 2^6$$

$$2^6 \times 2^4 = 2^{10}$$

Simplify $\frac{(5^3)^4}{5^{10}}$

Simplify $\frac{(5^3)^4}{5^{10}}$

$$(5^3)^4 = 5^{12}$$

$$\frac{5^{12}}{5^{10}} = 5^{12-10} = 5^2$$

$$3^n \times 3^5 = 3^{12}$$

Find the value of n

$$3^n \times 3^5 = 3^{12}$$

Find the value of n

The indices add, so $n + 5 = 12$.

$$n = 7$$

Write 8^4 as a power of 2

Write 8^4 as a power of 2

$$8 = 2^3, \text{ so } 8^4 = (2^3)^4.$$

$$8^4 = 2^{12}$$

Index rules with negative indices

True or false?

2^{-3} is a negative number

True or false?

2^{-3} is a negative number

False. A negative index gives a reciprocal, not a negative answer.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}, \text{ which is positive.}$$

$$2^3 = 8$$

↓

$$2^2 = 4$$

↓

$$2^1 = 2$$

↓

$$2^0 = 1$$

↓

halve each time

$$2^{-1} = \frac{1}{2}$$

↓

$$2^{-2} = \frac{1}{4}$$

↓

$$2^{-3} = \frac{1}{8}$$

True or false?

$$5^{-1} = \frac{1}{5}$$

True or false?

$$5^{-1} = \frac{1}{5}$$

True. An index of -1 gives the reciprocal of the base.

True or false?

$$4^{-2} = \frac{1}{8}$$

True or false?

$$4^{-2} = \frac{1}{8}$$

False. The -2 is an index, not a multiplier.

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

True or false?

$$2^4 \div 2^7 = 2^{-3}$$

True or false?

$$2^4 \div 2^7 = 2^{-3}$$

True. The indices are subtracted: $4 - 7 = -3$.

True or false?

$$2^{-3} \times 2^5 = 2^{-15}$$

True or false?

$$2^{-3} \times 2^5 = 2^{-15}$$

False. The indices are added, not multiplied.

$$2^{-3+5} = 2^2 = 4$$

Write 2^{-1} as a fraction

Write 2^{-1} as a fraction

$$2^{-1} = \frac{1}{2}$$

2^3	$= 8$	
		↓
2^2	$= 4$	
		↓
2^1	$= 2$	
		↓
2^0	$= 1$	
		↓
2^{-1}	$= \frac{1}{2}$	halve each time
		↓
2^{-2}	$= \frac{1}{4}$	
		↓
2^{-3}	$= \frac{1}{8}$	

Write 3^{-2} as a fraction

Write 3^{-2} as a fraction

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Write $\frac{1}{5^3}$ using a negative index

Write $\frac{1}{5^3}$ using a negative index

$$\frac{1}{5^3} = 5^{-3}$$

Write 10^{-3} as a decimal

Write 10^{-3} as a decimal

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

Simplify $2^3 \times 2^{-5}$

Simplify $2^3 \times 2^{-5}$

$$2^{3+(-5)} = 2^{-2}$$

Simplify $7^{-2} \div 7^3$

Simplify $7^{-2} \div 7^3$

$$7^{-2-3} = 7^{-5}$$

Simplify $(4^{-2})^3$

Simplify $(4^{-2})^3$

$$4^{-2 \times 3} = 4^{-6}$$

Work out $\frac{5^{-3}}{5^{-7}}$

Work out $\frac{5^{-3}}{5^{-7}}$

$$5^{-3-(-7)} = 5^{-3+7}$$

$$5^4 = 625$$

Work out $\left(\frac{2}{3}\right)^{-2}$

Work out $\left(\frac{2}{3}\right)^{-2}$

A negative index gives the reciprocal of the fraction.

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$2^n \times 2^{-5} = 2^{-1}$$

Find the value of n

$$2^n \times 2^{-5} = 2^{-1}$$

Find the value of n

The indices add, so $n - 5 = -1$.

$$n = 4$$

Simplifying expressions with index laws

True or false?

$$x^3 \times x^4 = x^{12}$$

True or false?

$$x^3 \times x^4 = x^{12}$$

False. The indices are added, not multiplied.

$$x^3 \times x^4 = x^7$$

True or false?

$$2a^3 \times 3a^2 = 6a^5$$

True or false?

$$2a^3 \times 3a^2 = 6a^5$$

True. The numbers are multiplied and the indices are added.

True or false?

$$(2x^3)^2 = 2x^6$$

True or false?

$$(2x^3)^2 = 2x^6$$

False. Everything inside the bracket is squared, including the 2.

$$(2x^3)^2 = 2^2 \times x^6 = 4x^6$$

True or false?

$$x^5 + x^3 = x^8$$

True or false?

$$x^5 + x^3 = x^8$$

False. Indices are added when terms are multiplied, not when they are added.

$x^5 + x^3$ cannot be simplified.

True or false?

$$\frac{12x^5}{3x^5} = 4$$

True or false?

$$\frac{12x^5}{3x^5} = 4$$

True. $12 \div 3 = 4$ and $x^5 \div x^5 = x^0 = 1$.

Simplify $x^3 \times x^4$

Simplify $x^3 \times x^4$

$$x^{3+4} = x^7$$

Simplify $y^8 \div y^3$

Simplify $y^8 \div y^3$

$$y^{8-3} = y^5$$

Simplify $3a^2 \times 4a^5$

Simplify $3a^2 \times 4a^5$

$$3 \times 4 = 12 \text{ and } a^{2+5} = a^7$$

$$12a^7$$

Simplify $\frac{20b^7}{5b^2}$

Simplify $\frac{20b^7}{5b^2}$

$$20 \div 5 = 4 \text{ and } b^{7-2} = b^5$$

$$4b^5$$

Simplify $(x^4)^3$

Simplify $(x^4)^3$

$$x^{4 \times 3} = x^{12}$$

Simplify $(3m^2)^3$

Simplify $(3m^2)^3$

The 3 is cubed as well as the m^2 .

$$3^3 \times m^6 = 27m^6$$

Simplify $5p^3q^2 \times 4p^2q^5$

Simplify $5p^3q^2 \times 4p^2q^5$

$$5 \times 4 = 20, p^{3+2} = p^5, q^{2+5} = q^7$$

$$20p^5q^7$$

Simplify $\frac{18x^6y^4}{6x^2y^4}$

Simplify $\frac{18x^6y^4}{6x^2y^4}$

$$18 \div 6 = 3, x^{6-2} = x^4, y^{4-4} = y^0 = 1$$
$$3x^4$$

Simplify $\frac{(2x^3)^4}{8x^5}$

Simplify $\frac{(2x^3)^4}{8x^5}$

$$(2x^3)^4 = 16x^{12}$$

$$\frac{16x^{12}}{8x^5} = 2x^7$$

Simplify $\frac{6a^5b^2 \times 3a^2b^4}{9a^4b^3}$

Simplify $\frac{6a^5b^2 \times 3a^2b^4}{9a^4b^3}$

$$6a^5b^2 \times 3a^2b^4 = 18a^7b^6$$

$$\frac{18a^7b^6}{9a^4b^3} = 2a^3b^3$$

Estimating roots and powers

True or false?

$\sqrt{50}$ lies between 7 and 8

True or false?

$\sqrt{50}$ lies between 7 and 8

True. $7^2 = 49$ and $8^2 = 64$, and 50 lies between those two square numbers.



True or false?

$$\sqrt{20} = 10$$

True or false?

$$\sqrt{20} = 10$$

False. Square rooting is not halving.

$10^2 = 100$, not 20. In fact $\sqrt{20}$ is between 4 and 5.

True or false?

$\sqrt{1000}$ is about 100

True or false?

$\sqrt{1000}$ is about 100

False. $100^2 = 10\,000$, not 1000.

$31^2 = 961$ and $32^2 = 1024$, so $\sqrt{1000}$ is about 31.6.

True or false?

$\sqrt[3]{30}$ lies between 3 and 4

True or false?

$\sqrt[3]{30}$ lies between 3 and 4

True. $3^3 = 27$ and $4^3 = 64$, and 30 lies between those two cube numbers.



True or false?

$\sqrt{0.64}$ is smaller than 0.64

True or false?

$\sqrt{0.64}$ is smaller than 0.64

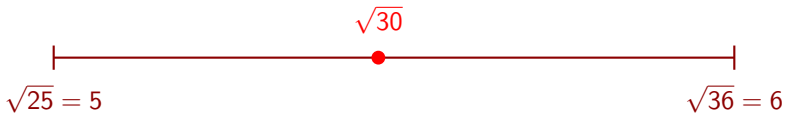
False. Square rooting only makes a number smaller when it is bigger than 1.

$0.8^2 = 0.64$, so $\sqrt{0.64} = 0.8$, which is larger than 0.64.

Between which two whole numbers
does $\sqrt{30}$ lie?

Between which two whole numbers
does $\sqrt{30}$ lie?

Between 5 and 6, because $5^2 = 25$ and $6^2 = 36$.



Between which two whole numbers
does $\sqrt{75}$ lie?

Between which two whole numbers
does $\sqrt{75}$ lie?

Between 8 and 9, because $8^2 = 64$ and $9^2 = 81$.



Estimate 3.9^3

Estimate 3.9^3

Round 3.9 to 4.

$4^3 = 64$, so 3.9^3 is about 64.

Between which two whole numbers
does $\sqrt[3]{50}$ lie?

Between which two whole numbers
does $\sqrt[3]{50}$ lie?

Between 3 and 4, because $3^3 = 27$ and $4^3 = 64$.



Estimate $\sqrt{17}$
to one decimal place

Estimate $\sqrt{17}$
to one decimal place

$\sqrt{16} = 4$, so the answer is just above 4.

$4.1^2 = 16.81$ and $4.2^2 = 17.64$, so $\sqrt{17} \approx 4.1$.

Estimate $\sqrt{90}$
to one decimal place

Estimate $\sqrt{90}$
to one decimal place

$\sqrt{81} = 9$ and $\sqrt{100} = 10$, so the answer is between 9 and 10.

$9.5^2 = 90.25$, so $\sqrt{90} \approx 9.5$.

Which is larger,
 $\sqrt{40}$ or 6.5?

Which is larger,
 $\sqrt{40}$ or 6.5?

Square both: $(\sqrt{40})^2 = 40$ and $6.5^2 = 42.25$.
6.5 is larger.

Estimate $\sqrt{2000}$
to the nearest whole number

Estimate $\sqrt{2000}$
to the nearest whole number

$$44^2 = 1936 \text{ and } 45^2 = 2025.$$

2000 is nearer to 2025, so $\sqrt{2000} \approx 45$.

Estimate $\frac{\sqrt{101}}{4.9}$

Estimate $\frac{\sqrt{101}}{4.9}$

Round each part: $\sqrt{101} \approx \sqrt{100} = 10$ and $4.9 \approx 5$.

$$\frac{10}{5} = 2$$

A square patio has an area of 150 m^2 .
Estimate the length of one side.

A square patio has an area of 150 m^2 .

Estimate the length of one side.

The side is $\sqrt{150}$.

$12^2 = 144$ and $13^2 = 169$, so the side is just over 12.

$12.2^2 = 148.84$, so the side is about 12.2 m.

Indices of the form $\frac{1}{a}$

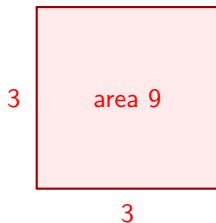
True or false?

$$9^{\frac{1}{2}} = 3$$

True or false?

$$9^{\frac{1}{2}} = 3$$

True. An index of $\frac{1}{2}$ means the square root, and $3^2 = 9$.



True or false?

$$9^{\frac{1}{2}} = 4.5$$

True or false?

$$9^{\frac{1}{2}} = 4.5$$

False. The index is not a multiplier, so this is not half of 9.

$$9^{\frac{1}{2}} = \sqrt{9} = 3$$

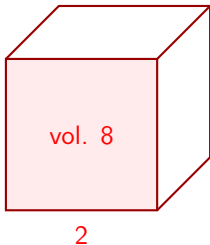
True or false?

$$8^{\frac{1}{3}} = 2$$

True or false?

$$8^{\frac{1}{3}} = 2$$

True. An index of $\frac{1}{3}$ means the cube root, and $2^3 = 8$.



True or false?

$$16^{\frac{1}{4}} = 4$$

True or false?

$$16^{\frac{1}{4}} = 4$$

False. An index of $\frac{1}{4}$ means the fourth root, not the square root.

$2^4 = 16$, so $16^{\frac{1}{4}} = 2$. It is $16^{\frac{1}{2}}$ that equals 4.

True or false?

$$100^{\frac{1}{3}} = 10$$

True or false?

$$100^{\frac{1}{3}} = 10$$

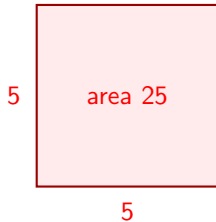
False. $10^3 = 1000$, not 100.

$\sqrt[3]{100}$ is about 4.6. It is $100^{\frac{1}{2}}$ that equals 10.

Work out $25^{\frac{1}{2}}$

Work out $25^{\frac{1}{2}}$

$$25^{\frac{1}{2}} = \sqrt{25} = 5$$



Work out $64^{\frac{1}{2}}$

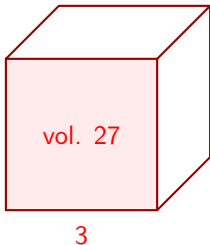
Work out $64^{\frac{1}{2}}$

$$\sqrt{64} = 8, \text{ because } 8^2 = 64.$$

Work out $27^{\frac{1}{3}}$

Work out $27^{\frac{1}{3}}$

$\sqrt[3]{27} = 3$, because $3^3 = 27$.



Work out $81^{\frac{1}{4}}$

Work out $81^{\frac{1}{4}}$

The fourth root: $3 \times 3 \times 3 \times 3 = 81$.

$$81^{\frac{1}{4}} = 3$$

Work out $1000^{\frac{1}{3}}$

Work out $1000^{\frac{1}{3}}$

$$\sqrt[3]{1000} = 10, \text{ because } 10^3 = 1000.$$

Write $\sqrt[5]{x}$ using an index

Write $\sqrt[5]{x}$ using an index

$$\sqrt[5]{x} = x^{\frac{1}{5}}$$

Work out $\left(\frac{1}{16}\right)^{\frac{1}{2}}$

Work out $\left(\frac{1}{16}\right)^{\frac{1}{2}}$

Square root the numerator and the denominator.

$$\frac{\sqrt{1}}{\sqrt{16}} = \frac{1}{4}$$

Work out $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

Work out $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

Cube root the numerator and the denominator.

$$\frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

Work out $0.04^{\frac{1}{2}}$

Work out $0.04^{\frac{1}{2}}$

$$0.2 \times 0.2 = 0.04$$

$$0.04^{\frac{1}{2}} = 0.2$$

Work out $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

Work out $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

The fifth root of 32 is 2, because $2^5 = 32$.

$$\frac{1}{2}$$

Indices of the form $\frac{a}{b}$

True or false?

$$8^{\frac{2}{3}} = 4$$

True or false?

$$8^{\frac{2}{3}} = 4$$

True. The denominator of the index gives the root and the numerator gives the power.



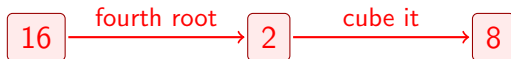
True or false?

$$16^{\frac{3}{4}} = 12$$

True or false?

$$16^{\frac{3}{4}} = 12$$

False. This is $\frac{3}{4}$ of 16, not 16 to the power $\frac{3}{4}$.



True or false?

$$25^{\frac{3}{2}} = 75$$

True or false?

$$25^{\frac{3}{2}} = 75$$

False. The $\frac{3}{2}$ is an index, not a multiplier.

$$\sqrt{25} = 5, \text{ then } 5^3 = 125.$$

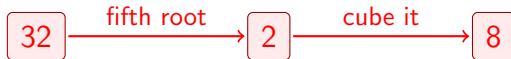
True or false?

To work out $32^{\frac{3}{5}}$ you can find $\sqrt[5]{32}$ first,
then cube the answer

True or false?

To work out $32^{\frac{3}{5}}$ you can find $\sqrt[5]{32}$ first,
then cube the answer

True. Finding the root first keeps the numbers small.



True or false?

$4^{\frac{3}{2}}$ and $4^{1.5}$ mean the same thing

True or false?

$4^{\frac{3}{2}}$ and $4^{1.5}$ mean the same thing

True. $\frac{3}{2} = 1.5$, so both are $\sqrt{4}$ cubed, which is 8.

Work out $4\frac{3}{2}$

Work out $4^{\frac{3}{2}}$



Work out $8^{\frac{2}{3}}$

Work out $8^{\frac{2}{3}}$



Work out $9^{\frac{3}{2}}$

Work out $9^{\frac{3}{2}}$



Work out $16^{\frac{3}{4}}$

Work out $16^{\frac{3}{4}}$



Work out $27^{\frac{2}{3}}$

Work out $27^{\frac{2}{3}}$



Work out $32^{\frac{3}{5}}$

Work out $32^{\frac{3}{5}}$



Work out $125^{\frac{4}{3}}$

Work out $125^{\frac{4}{3}}$



Work out $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Work out $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

$$\left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

Work out $0.25^{\frac{3}{2}}$

Work out $0.25^{\frac{3}{2}}$

$$\sqrt{0.25} = 0.5, \text{ because } 0.5^2 = 0.25.$$

$$0.5^3 = 0.125$$

Work out $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

Work out $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

Simplifying expressions with negative fractional indices

True or false?

$$9^{-\frac{1}{2}} = \frac{1}{3}$$

True or false?

$$9^{-\frac{1}{2}} = \frac{1}{3}$$

True. The $\frac{1}{2}$ gives the square root and the minus sign gives the reciprocal.



True or false?

$$9^{-\frac{1}{2}} = -3$$

True or false?

$$9^{-\frac{1}{2}} = -3$$

False. A negative index gives a reciprocal, not a negative answer.

$$9^{-\frac{1}{2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

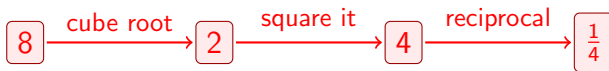
True or false?

$$8^{-\frac{2}{3}} = \frac{1}{4}$$

True or false?

$$8^{-\frac{2}{3}} = \frac{1}{4}$$

True. Cube root, then square, then take the reciprocal.



True or false?

$$16^{-\frac{3}{4}} = \frac{1}{12}$$

True or false?

$$16^{-\frac{3}{4}} = \frac{1}{12}$$

False. The $\frac{3}{4}$ is an index, not a multiplier.

$\sqrt[4]{16} = 2$, then $2^3 = 8$, so the answer is $\frac{1}{8}$.

True or false?

$25^{-\frac{1}{2}}$ is smaller than $25^{-\frac{3}{2}}$

True or false?

$25^{-\frac{1}{2}}$ is smaller than $25^{-\frac{3}{2}}$

False. The more negative the index, the smaller the result for a base bigger than 1.

$$25^{-\frac{1}{2}} = \frac{1}{5} = 0.2 \text{ and } 25^{-\frac{3}{2}} = \frac{1}{125} = 0.008.$$

Work out $4^{-\frac{1}{2}}$

Work out $4^{-\frac{1}{2}}$

$$4^{-\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

Work out $8^{-\frac{1}{3}}$

Work out $8^{-\frac{1}{3}}$

$$8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$$

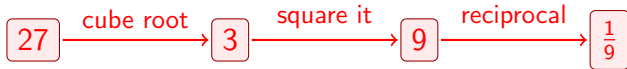
Work out $9^{-\frac{1}{2}}$

Work out $9^{-\frac{1}{2}}$

$$9^{-\frac{1}{2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

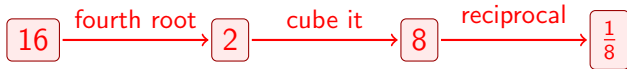
Work out $27^{-\frac{2}{3}}$

Work out $27^{-\frac{2}{3}}$



Work out $16^{-\frac{3}{4}}$

Work out $16^{-\frac{3}{4}}$



Work out $\left(\frac{1}{-4}\right)^{-\frac{1}{2}}$

Work out $\left(\frac{1}{-4}\right)^{-\frac{1}{2}}$

Take the reciprocal of the fraction, then square root.

$$4^{\frac{1}{2}} = 2$$

Work out $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

Work out $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

Take the reciprocal of the fraction, then square root.

$$\left(\frac{16}{9}\right)^{\frac{1}{2}} = \frac{4}{3}$$

Work out $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

Work out $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

$$\begin{aligned}\left(\frac{8}{27}\right)^{-\frac{2}{3}} &= \left(\frac{27}{8}\right)^{\frac{2}{3}} \\ \left(\frac{27}{8}\right)^{\frac{2}{3}} &= \left(\frac{3}{2}\right)^2 \\ \left(\frac{3}{2}\right)^2 &= \frac{9}{4}\end{aligned}$$

Simplify $\frac{x^{\frac{1}{2}}}{x^{-\frac{3}{2}}}$

Simplify $\frac{x^{\frac{1}{2}}}{x^{-\frac{3}{2}}}$

The indices are subtracted.

$$x^{\frac{1}{2} - (-\frac{3}{2})} = x^2$$

Simplify $(16x^8)^{-\frac{3}{4}}$

Simplify $(16x^8)^{-\frac{3}{4}}$

$$(16x^8)^{\frac{1}{4}} = 2x^2$$

$$(2x^2)^3 = 8x^6$$

$$(16x^8)^{-\frac{3}{4}} = \frac{1}{8x^6}$$