

Factors, Multiples and Primes: Quick Questions

Mini whiteboard questions, with solutions

Contents

dzielniki

- Factors

wielokrotności

- Multiples

liczby pierwsze

- Primes

liczba pierwsza

- Is it prime?

Rozkład na czynniki pierwsze

- Prime factor decomposition

najmniejszy wspólny dzielnik i wielokrotność

- HCF and LCM from lists

najmniejszy wspólny dzielnik i wielokrotność z rozkładem na czynniki pierwsze

- HCF and LCM by prime factor decomposition

Liczby kwadratowe i czynniki pierwsze

- Square numbers from prime factors

najmniejszy wspólny dzielnik i wielokrotność

- Problem solving with HCF and LCM

Factors

dzielniki

True or false?

7 is a ^{dzielnik} factor of 21

True or false?

7 is a ^{dzielnik} factor of 21

True. $21 = 7 \times 3$, so 21 divides exactly by 7.

True or false?

1 is a ^{dzielnik} factor of every whole number

True or false?

1 is a ^{dzielnik} factor of every whole number

True. Every number n can be written as $1 \times n$.

True or false?

6 is a ^{dzielnik} factor of 16

True or false?

6 is a ^{dzielnik} factor of 16

False. ^{dzielniki} The factors of 16 are 1, 2, 4, 8, 16;

16 ÷ 6 is not a whole number.

True or false?

Every number has an even number of ^{dzielniki} factors

True or false?

Every number has an even number of ^{dzielniki} factors

^{Dzielniki}
False. Factors usually pair up, but 9 has just 1, 3, 9 — three

^{dzielniki} factors — because 3×3 uses the same ^{dzielnik} factor twice.

True or false?

If 4 is a ^{dzielnik}factor of a number,

then 8 must be a ^{dzielnik}factor of it too

True or false?

If 4 is a ^{dzielnik}factor of a number,

then 8 must be a ^{dzielnik}factor of it too

False. 4 is a ^{dzielnik}factor of 12, but 8 is not.

List all the ^{dzielniki} factors of 12

List all the ^{dzielniki} factors of 12

1, 2, 3, 4, 6, 12

List all the ^{dzielniki} factors of 20

List all the ^{dzielniki} factors of 20

1, 2, 4, 5, 10, 20

List all the ^{dzielniki} factors of 36

List all the ^{dzielniki} factors of 36

1, 2, 3, 4, 6, 9, 12, 18, 36

How many ^{dzielniki} factors does 16 have?

How many ^{dzielniki} factors does 16 have?

1, 2, 4, 8, 16

^{dzielniki}
Five factors.

Find the ^{największy wspólny dzielnik} highest common factor of 12 and 18

Find the ^{największy wspólny dzielnik} highest common factor of 12 and 18

^{Dzielniki}
Factors of 12: 1, 2, 3, 4, **6**, 12

^{Dzielniki}
Factors of 18: 1, 2, 3, **6**, 9, 18

$$\text{HCF} = 6$$

Find the ^{największy wspólny dzielnik} highest common factor of 24 and 36

Find the ^{największy wspólny dzielnik} highest common factor of 24 and 36

^{Dzielniki}
Factors of 24: 1, 2, 3, 4, 6, 8, **12**, 24

^{Dzielniki}
Factors of 36: 1, 2, 3, 4, 6, 9, **12**, 18, 36

$$\text{HCF} = 12$$

List all the ^{pary czynników} factor pairs of 48

List all the ^{pary czynników} factor pairs of 48

$$1 \times 48$$

$$2 \times 24$$

$$3 \times 16$$

$$4 \times 12$$

$$6 \times 8$$

Which whole number below 30 has the most ^{dzielniki} factors?

Which whole number below 30 has the most ^{dzielniki} factors?

24, with eight ^{dzielniki} factors:

1, 2, 3, 4, 6, 8, 12, 24

n has exactly three ^{dzielniki} factors.

Give a possible value of n .

n has exactly three ^{dzielniki} factors.

Give a possible value of n .

4 (1, 2, 4), 9 (1, 3, 9) or 25 (1, 5, 25).

Any ^{liczba pierwsza} prime squared works.

Explain why liczby kwadratowe **square numbers** have an odd number of dzielniki **factors**

liczby kwadratowe dzielniki
Explain why **square numbers** have an odd number of **factors**

Dzielniki pierwiastek
Factors come in pairs, one either side of the **square root**.

liczba kwadratowa pierwiastek
In a **square number** the **square root** pairs with itself, so one
dzielnik
factor is left unpaired.

Wielokrotności Multiples

True or false?

24 is a ^{wielokrotność} multiple of 6

True or false?

24 is a ^{wielokrotność} multiple of 6

True. $6 \times 4 = 24$.

True or false?

Every ^{wielokrotność} multiple of 10 is also a ^{wielokrotność} multiple of 5

True or false?

Every ^{wielokrotność} multiple of 10 is also a ^{wielokrotność} multiple of 5

True. $10 = 5 \times 2$, so every ^{wielokrotność} multiple of 10 already contains
^{dzielnik} a factor of 5.

True or false?

35 is a ^{wielokrotność} multiple of 4

True or false?

35 is a ^{wielokrotność} multiple of 4

False. $4 \times 8 = 32$ and $4 \times 9 = 36$, so 35 is stepped over.

True or false?

Every ^{wielokrotność} multiple of 3 is also a ^{wielokrotność} multiple of 9

True or false?

Every ^{wielokrotność} multiple of 3 is also a ^{wielokrotność} multiple of 9

False. 6 is a ^{wielokrotność} multiple of 3 but not of 9.

The other way round *is* true.

True or false?

The ^{najmniejsza wspólna wielokrotność} **lowest common multiple** of two numbers is always
found by multiplying them together

True or false?

The ^{najmniejsza wspólna wielokrotność}
lowest common multiple of two numbers is always
found by multiplying them together

False. $4 \times 6 = 24$, but the ^{najmniejsza wspólna wielokrotność}
LCM of 4
and 6 is 12.

Multiplying only works when the numbers share no ^{dzielniki}
factors.

Write down the first five ^{wielokrotności} multiples of 4

Write down the first five ^{wielokrotności} multiples of 4

4, 8, 12, 16, 20

Write down the first five ^{wielokrotności} multiples of 7

Write down the first five ^{wielokrotności} multiples of 7

7, 14, 21, 28, 35

Write down the ^{wielokrotności} multiples of 6 between 20 and 50

Write down the ^{wielokrotności} multiples of 6 between 20 and 50

24, 30, 36, 42, 48

Is 126 a ^{wielokrotność} multiple of 9?

Is 126 a ^{wielokrotność} multiple of 9?

Yes. $1 + 2 + 6 = 9$, and $9 \times 14 = 126$.

Find the ^{najmniejsza wspólna wielokrotność} lowest common multiple of 4 and 6

Find the najmniejsza wspólna wielokrotność **lowest common multiple** of 4 and 6

4, 8, **12**, 16, ...

6, **12**, 18, ...

LCM = 12

Find the najmniejsza wspólna wielokrotność **lowest common multiple** of 8 and 12

Find the najmniejsza wspólna wielokrotność **lowest common multiple** of 8 and 12

8, 16, **24**, 32, ...

12, **24**, 36, ...

LCM = 24

Find the najmniejsza wspólna wielokrotność **lowest common multiple** of 5, 6 and 9

Find the najmniejsza wspólna wielokrotność **lowest common multiple** of 5, 6 and 9

najmniejsza wspólna wielokrotność
LCM of 5 and 6 is 30.

najmniejsza wspólna wielokrotność
LCM of 30 and 9 is 90.

$$\text{LCM} = 90$$

A number is a ^{wielokrotność} **multiple** of both 4 and 10.

What is the smallest it could be?

A number is a ^{wielokrotność} **multiple** of both 4 and 10.

What is the smallest it could be?

20, the ^{najmniejsza wspólna wielokrotność} **lowest common multiple** of 4 and 10.

Two lighthouses flash together now.
One flashes every 12 seconds, the other every 18 seconds.
When do they next flash together?

Two lighthouses flash together now.
One flashes every 12 seconds, the other every 18 seconds.
When do they next flash together?

$$\text{LCM}(12, 18) = 36$$

In 36 seconds.

Find the smallest number above 1 that leaves a remainder of 1 when divided by 4, by 5 and by 6

Find the smallest number above 1 that leaves a remainder of 1 when divided by 4, by 5 and by 6

The number must be one more than a common ^{wielokrotność} multiple of

4, 5 and 6.

$\text{LCM}(4, 5, 6) = 60$, so the number is 61.

Primes

liczby pierwsze

True or false?

1 is a ^{liczba pierwsza}
prime number

True or false?

1 is a ^{liczba pierwsza}
prime number

False. A ^{liczba pierwsza}
prime has exactly two ^{dzielniki}
factors, but 1 has only
one.

True or false?

2 is the only even ^{liczba pierwsza} prime number

True or false?

2 is the only even ^{liczba pierwsza} prime number

True. Every other even number has 2 as an extra ^{dzielnik} factor, so
it has more than two ^{dzielniki} factors.

True or false?

51 is a ^{liczba pierwsza}
prime number

True or false?

51 is a ^{liczba pierwsza}
prime number

False. $51 = 3 \times 17$.

Its digits add to 6, so it divides by 3.

True or false?

There are infinitely many ^{liczby pierwsze}
prime numbers

True or false?

There are infinitely many ^{liczby pierwsze} **prime** numbers

True. The ^{liczby pierwsze} **primes** never run out, although they do thin out as the numbers get larger.

True or false?

If a number ends in 3, it must be ^{liczba pierwsza}
prime

True or false?

If a number ends in 3, it must be ^{liczba pierwsza}prime

False. $33 = 3 \times 11$ and $63 = 7 \times 9$.

Write down all the ^{liczby pierwsze}
prime numbers between 1
and 10

Write down all the ^{liczby pierwsze}
prime numbers between 1

and 10

2, 3, 5, 7

Write down all the ^{liczby pierwsze}
prime numbers between 10
and 20

Write down all the ^{liczby pierwsze}
prime numbers between 10
and 20

11, 13, 17, 19

Is 27 a ^{liczba pierwsza}
prime number?

Explain your answer.

Is 27 a ^{liczba pierwsza}
prime number?

Explain your answer.

No. $27 = 3 \times 9$, so it has more than two ^{dzielniki}
factors.

Write down the first ^{liczba pierwsza}
prime number greater than

30

Write down the first ^{liczba pierwsza}
prime number greater than

30

31

Is 91 a ^{liczba pierwsza}
prime number?

Is 91 a ^{liczba pierwsza}
prime number?

No. $91 = 7 \times 13$.

How many ^{liczby pierwsze}
prime numbers are there between 20
and 40?

How many ^{liczby pierwsze}
prime numbers are there between 20
and 40?

23, 29, 31, 37

Four of them.

Is 143 a ^{liczba pierwsza}
prime number?

Is 143 a ^{liczba pierwsza}
prime number?

No. $143 = 11 \times 13.$

Find two ^{liczby pierwsze}
prime numbers that add up to 24

Find two ^{liczby pierwsze}
prime numbers that add up to 24

$$5 + 19 \quad 7 + 17 \quad 11 + 13$$

To test whether 211 is ^{liczba pierwsza}
prime ,

which numbers do you need to divide by?

Is it ^{liczba pierwsza}
prime ?

To test whether 211 is ^{liczba pierwsza}prime ,

which numbers do you need to divide by?

Is it ^{liczba pierwsza}prime ?

Only the ^{liczby pierwsze}primes up to $\sqrt{211} \approx 14.5$:

2, 3, 5, 7, 11, 13

None of them divides 211, so 211 is ^{liczba pierwsza}prime .

Explain why $n^2 - 1$ can never be liczba pierwsza
prime when $n > 2$

Explain why $n^2 - 1$ can never be ^{liczba pierwsza}
^{prime} when $n > 2$

$$n^2 - 1 = (n - 1)(n + 1)$$

When $n > 2$ both brackets are bigger than 1, so the number

always has more than two ^{dzielniki}
^{factors}.

Is it ^{liczba pierwsza} prime?

1

1

Factors of 1: 1

1 has only one **factor** . A **prime** number has exactly two **factors** , 1 and itself, and for 1 those would be the same **factor** , so 1 is not a **prime** number.

2

2

Factors of 2: 1, 2

2 has exactly two **dzielniki** factors , 1 and itself, so 2 is a **liczba pierwsza** prime number. It is the only even **liczba pierwsza** prime number, because every other even number has 2 as a third **dzielnik** factor .

3

3

Factors of 3: 1, 3

3 has exactly two ^{dzielniki} factors , 1 and itself, so 3 is a ^{liczba pierwsza} prime number.

4

4

Factors of 4: 1, 2, 4

4 has three ^{dzielniki} factors, not two, so 4 is not a ^{liczba pierwsza} prime number.

5

5

Factors of 5: 1, 5

5 has exactly two **dzielniki** factors , 1 and itself, so 5 is a **liczba pierwsza** prime number.

We only need to test 2 as a **dzielnik** factor , because $3 \times 3 = 9$ is bigger than 5.

6

6

Factors of 6: 1, 2, 3, 6

dzielniki

liczba pierwsza

6 has four factors , not two, so 6 is not a prime number.

7

7

Factors of 7: 1, 7

7 has exactly two **dzielniki** factors , 1 and itself, so 7 is a **liczba pierwsza** prime number.

We only need to test 2 as a **dzielnik** factor , because $3 \times 3 = 9$ is bigger than 7.

8

8

Factors of 8: 1, 2, 4, 8

dzielniki

8 has four factors , not two, so 8 is not a

liczba pierwsza

prime

number.

9

9

Factors of 9: 1, 3, 9

dzielniki

9 has three factors , not two, so 9 is not a

liczba pierwsza

prime

number.

10

10

Factors of 10: 1, 2, 5, 10

dzielniki

liczba pierwsza

10 has four factors , not two, so 10 is not a prime number.

11

11

Factors of 11: 1, 11

11 has exactly two ^{dzielniki} factors, 1 and itself, so 11 is a ^{liczba pierwsza} prime number.

We only need to test 2 and 3 as ^{dzielniki} factors, because $5 \times 5 = 25$ is bigger than 11.

12

12

Factors of 12: 1, 2, 3, 4, 6, 12

dzielniki

liczba pierwsza

12 has six factors , not two, so 12 is not a prime number.

13

13

Factors of 13: 1, 13

13 has exactly two dzielniki factors, 1 and itself, so 13 is a liczba pierwsza prime number.

We only need to test 2 and 3 as dzielniki factors, because $5 \times 5 = 25$ is bigger than 13.

14

14

Factors of 14: 1, 2, 7, 14

dzielniki

liczba pierwsza

14 has four factors , not two, so 14 is not a prime number.

15

15

Factors of 15: 1, 3, 5, 15

dzielniki

liczba pierwsza

15 has four factors , not two, so 15 is not a prime number.

16

16

Factors of 16: 1, 2, 4, 8, 16

16 has five ^{dzielniki} factors , not two, so 16 is not a ^{liczba pierwsza} prime number.

17

17

Factors of 17: 1, 17

17 has exactly two dzielniki factors, 1 and itself, so 17 is a liczba pierwsza prime number.

We only need to test 2 and 3 as dzielniki factors, because $5 \times 5 = 25$ is bigger than 17.

18

18

Factors of 18: 1, 2, 3, 6, 9, 18

dzielniki

liczba pierwsza

18 has six factors , not two, so 18 is not a prime number.

19

19

Factors of 19: 1, 19

19 has exactly two dzielniki factors, 1 and itself, so 19 is a liczba pierwsza prime number.

We only need to test 2 and 3 as dzielniki factors, because $5 \times 5 = 25$ is bigger than 19.

20

20

Factors of 20: 1, 2, 4, 5, 10, 20

dzielniki

liczba pierwsza

20 has six factors , not two, so 20 is not a prime number.

21

21

Factors of 21: 1, 3, 7, 21

dzielniki

liczba pierwsza

21 has four factors , not two, so 21 is not a prime number.

22

22

Factors of 22: 1, 2, 11, 22

dzielniki

liczba pierwsza

22 has four factors , not two, so 22 is not a prime number.

23

23

Factors of 23: 1, 23

23 has exactly two dzielniki factors, 1 and itself, so 23 is a liczba pierwsza prime number.

We only need to test 2 and 3 as dzielniki factors, because $5 \times 5 = 25$ is bigger than 23.

24

24

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

24 has eight ^{dzielniki} factors , not two, so 24 is not a ^{liczba pierwsza} prime number.

25

25

Factors of 25: 1, 5, 25

dzielniki

liczba pierwsza

25 has three factors , not two, so 25 is not a prime number.

26

26

Factors of 26: 1, 2, 13, 26

26 has four ^{dzielniki} factors , not two, so 26 is not a ^{liczba pierwsza} prime number.

27

27

Factors of 27: 1, 3, 9, 27

dzielniki

27 has four factors , not two, so 27 is not a

liczba pierwsza

prime

number.

28

28

Factors of 28: 1, 2, 4, 7, 14, 28

dzielniki

liczba pierwsza

28 has six factors , not two, so 28 is not a prime number.

29

29

Factors of 29: 1, 29

29 has exactly two dzielniki factors, 1 and itself, so 29 is a liczba pierwsza prime number.

We only need to test 2, 3 and 5 as dzielniki factors, because $7 \times 7 = 49$ is bigger than 29.

30

30

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

dzielniki

liczba pierwsza

30 has eight factors , not two, so 30 is not a prime number.

31

31

Factors of 31: 1, 31

31 has exactly two ^{dzielniki} factors, 1 and itself, so 31 is a ^{liczba pierwsza} prime number.

We only need to test 2, 3 and 5 as ^{dzielniki} factors, because $7 \times 7 = 49$ is bigger than 31.

32

32

Factors of 32: 1, 2, 4, 8, 16, 32

dzielniki

liczba pierwsza

32 has six factors , not two, so 32 is not a prime number.

33

33

Factors of 33: 1, 3, 11, 33

dzielniki

33 has four factors , not two, so 33 is not a

liczba pierwsza

prime

number.

34

34

Factors of 34: 1, 2, 17, 34

34 has four dzielniki factors , not two, so 34 is not a liczba pierwsza prime number.

35

35

Factors of 35: 1, 5, 7, 35

dzielniki

liczba pierwsza

35 has four factors , not two, so 35 is not a prime number.

36

36

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

dzielniki

liczba pierwsza

36 has nine factors , not two, so 36 is not a prime number.

37

37

Factors of 37: 1, 37

37 has exactly two ^{dzielniki} factors , 1 and itself, so 37 is a ^{liczba pierwsza} prime number.

We only need to test 2, 3 and 5 as ^{dzielniki} factors , because $7 \times 7 = 49$ is bigger than 37.

38

38

Factors of 38: 1, 2, 19, 38

dzielniki

38 has four factors , not two, so 38 is not a

liczba pierwsza

prime

number.

39

39

Factors of 39: 1, 3, 13, 39

dzielniki

39 has four factors , not two, so 39 is not a

liczba pierwsza

prime

number.

40

40

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

dzielniki

liczba pierwsza

40 has eight factors , not two, so 40 is not a prime number.

41

41

Factors of 41: 1, 41

41 has exactly two ^{dzielniki} factors , 1 and itself, so 41 is a ^{liczba pierwsza} prime number.

We only need to test 2, 3 and 5 as ^{dzielniki} factors , because $7 \times 7 = 49$ is bigger than 41.

42

42

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

42 has eight ^{dzielniki} factors , not two, so 42 is not a ^{liczba pierwsza} prime number.

43

43

Factors of 43: 1, 43

43 has exactly two ^{dzielniki} factors , 1 and itself, so 43 is a ^{liczba pierwsza} prime number.

We only need to test 2, 3 and 5 as ^{dzielniki} factors , because $7 \times 7 = 49$ is bigger than 43.

44

44

Factors of 44: 1, 2, 4, 11, 22, 44

dzielniki
44 has six factors , not two, so 44 is not a liczba pierwsza prime number.

45

45

Factors of 45: 1, 3, 5, 9, 15, 45

dzielniki

liczba pierwsza

45 has six factors , not two, so 45 is not a prime number.

46

46

Factors of 46: 1, 2, 23, 46

46 has four ^{dzielniki} factors , not two, so 46 is not a ^{liczba pierwsza} prime number.

47

47

Factors of 47: 1, 47

47 has exactly two ^{dzielniki} factors , 1 and itself, so 47 is a ^{liczba pierwsza} prime number.

We only need to test 2, 3 and 5 as ^{dzielniki} factors , because $7 \times 7 = 49$ is bigger than 47.

48

48

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

dzielniki

liczba pierwsza

48 has ten **factors** , not two, so 48 is not a **prime** number.

49

49

Factors of 49: 1, 7, 49

dzielniki

liczba pierwsza

49 has three factors , not two, so 49 is not a prime number.

50

50

Factors of 50: 1, 2, 5, 10, 25, 50

dzielniki

liczba pierwsza

50 has six factors , not two, so 50 is not a prime number.

51

51

Factors of 51: 1, 3, 17, 51

dzielniki

liczba pierwsza

51 has four factors , not two, so 51 is not a prime number.

52

52

Factors of 52: 1, 2, 4, 13, 26, 52

dzielniki

liczba pierwsza

52 has six factors , not two, so 52 is not a prime number.

53

53

Factors of 53: 1, 53

53 has exactly two **dzielniki** factors, 1 and itself, so 53 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors, because $11 \times 11 = 121$ is bigger than 53.

54

54

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

54 has eight ^{dzielniki} factors , not two, so 54 is not a ^{liczba pierwsza} prime number.

55

55

Factors of 55: 1, 5, 11, 55

dzielniki

liczba pierwsza

55 has four factors , not two, so 55 is not a prime number.

56

56

Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

dzielniki

liczba pierwsza

56 has eight factors , not two, so 56 is not a prime number.

57

57

Factors of 57: 1, 3, 19, 57

dzielniki

liczba pierwsza

57 has four factors , not two, so 57 is not a prime number.

58

58

Factors of 58: 1, 2, 29, 58

dzielniki
58 has four factors , not two, so 58 is not a liczba pierwsza
prime number.

59

59

Factors of 59: 1, 59

59 has exactly two **dzielniki** factors, 1 and itself, so 59 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors, because $11 \times 11 = 121$ is bigger than 59.

60

60

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

dzielniki

liczba pierwsza

60 has twelve **factors** , not two, so 60 is not a **prime** number.

61

61

Factors of 61: 1, 61

61 has exactly two dzielniki factors, 1 and itself, so 61 is a liczba pierwsza prime number.

We only need to test 2, 3, 5 and 7 as dzielniki factors, because $11 \times 11 = 121$ is bigger than 61.

62

62

Factors of 62: 1, 2, 31, 62

dzielniki

liczba pierwsza

62 has four factors , not two, so 62 is not a prime number.

63

63

Factors of 63: 1, 3, 7, 9, 21, 63

dzielniki

liczba pierwsza

63 has six factors , not two, so 63 is not a prime number.

64

64

Factors of 64: 1, 2, 4, 8, 16, 32, 64

64 has seven dzielniki factors , not two, so 64 is not a liczba pierwsza prime number.

65

65

Factors of 65: 1, 5, 13, 65

dzielniki

65 has four factors , not two, so 65 is not a

liczba pierwsza

prime

number.

66

66

Factors of 66: 1, 2, 3, 6, 11, 22, 33, 66

dzielniki

liczba pierwsza

66 has eight factors , not two, so 66 is not a prime number.

67

67

Factors of 67: 1, 67

67 has exactly two **dzielniki** factors, 1 and itself, so 67 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors, because $11 \times 11 = 121$ is bigger than 67.

68

68

Factors of 68: 1, 2, 4, 17, 34, 68

dzielniki

liczba pierwsza

68 has six factors , not two, so 68 is not a prime number.

69

69

Factors of 69: 1, 3, 23, 69

dzielniki

69 has four factors , not two, so 69 is not a

liczba pierwsza

prime

number.

70

70

Factors of 70: 1, 2, 5, 7, 10, 14, 35, 70

dzielniki

liczba pierwsza

70 has eight **factors** , not two, so 70 is not a **prime** number.

71

71

Factors of 71: 1, 71

71 has exactly two dzielniki factors , 1 and itself, so 71 is a liczba pierwsza prime number.

We only need to test 2, 3, 5 and 7 as dzielniki factors , because $11 \times 11 = 121$ is bigger than 71.

72

72

Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

dzielniki

liczba pierwsza

72 has twelve **factors** , not two, so 72 is not a **prime** number.

73

73

Factors of 73: 1, 73

73 has exactly two dzielniki factors , 1 and itself, so 73 is a liczba pierwsza prime number.

We only need to test 2, 3, 5 and 7 as dzielniki factors , because $11 \times 11 = 121$ is bigger than 73.

74

74

Factors of 74: 1, 2, 37, 74

dzielniki

liczba pierwsza

74 has four factors , not two, so 74 is not a prime number.

75

75

Factors of 75: 1, 3, 5, 15, 25, 75

dzielniki

liczba pierwsza

75 has six factors , not two, so 75 is not a prime number.

76

76

Factors of 76: 1, 2, 4, 19, 38, 76

dzielniki

liczba pierwsza

76 has six factors , not two, so 76 is not a prime number.

77

77

Factors of 77: 1, 7, 11, 77

dzielniki

liczba pierwsza

77 has four factors , not two, so 77 is not a prime number.

78

78

Factors of 78: 1, 2, 3, 6, 13, 26, 39, 78

dzielniki

liczba pierwsza

78 has eight **factors** , not two, so 78 is not a **prime** number.

79

79

Factors of 79: 1, 79

79 has exactly two **dzielniki** factors , 1 and itself, so 79 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors , because $11 \times 11 = 121$ is bigger than 79.

80

80

Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

dzielniki

liczba pierwsza

80 has ten **factors** , not two, so 80 is not a **prime** number.

81

81

Factors of 81: 1, 3, 9, 27, 81

dzielniki

liczba pierwsza

81 has five factors , not two, so 81 is not a prime number.

82

82

Factors of 82: 1, 2, 41, 82

dzielniki

82 has four factors , not two, so 82 is not a

liczba pierwsza

prime

number.

83

83

Factors of 83: 1, 83

83 has exactly two **dzielniki** factors, 1 and itself, so 83 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors, because $11 \times 11 = 121$ is bigger than 83.

84

84

Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

dzielniki

liczba pierwsza

84 has twelve **factors** , not two, so 84 is not a **prime** number.

85

85

Factors of 85: 1, 5, 17, 85

dzielniki

liczba pierwsza

85 has four factors , not two, so 85 is not a prime number.

86

86

Factors of 86: 1, 2, 43, 86

dzielniki

86 has four factors , not two, so 86 is not a

liczba pierwsza

prime

number.

87

87

Factors of 87: 1, 3, 29, 87

dzielniki

87 has four factors , not two, so 87 is not a

liczba pierwsza

prime

number.

88

88

Factors of 88: 1, 2, 4, 8, 11, 22, 44, 88

dzielniki

liczba pierwsza

88 has eight factors , not two, so 88 is not a prime number.

89

89

Factors of 89: 1, 89

89 has exactly two **dzielniki** factors, 1 and itself, so 89 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors, because $11 \times 11 = 121$ is bigger than 89.

90

90

Factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

dzielniki

liczba pierwsza

90 has twelve **factors** , not two, so 90 is not a **prime** number.

91

91

Factors of 91: 1, 7, 13, 91

dzielniki

liczba pierwsza

91 has four factors , not two, so 91 is not a prime number.

92

92

Factors of 92: 1, 2, 4, 23, 46, 92

dzielniki

liczba pierwsza

92 has six factors , not two, so 92 is not a prime number.

93

93

Factors of 93: 1, 3, 31, 93

dzielniki

liczba pierwsza

93 has four factors , not two, so 93 is not a prime number.

94

94

Factors of 94: 1, 2, 47, 94

dzielniki

liczba pierwsza

94 has four factors , not two, so 94 is not a prime number.

95

95

Factors of 95: 1, 5, 19, 95

dzielniki

95 has four factors , not two, so 95 is not a

liczba pierwsza

prime

number.

96

96

Factors of 96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

dzielniki

liczba pierwsza

96 has twelve **factors** , not two, so 96 is not a **prime** number.

97

97

Factors of 97: 1, 97

97 has exactly two **dzielniki** factors , 1 and itself, so 97 is a **liczba pierwsza** prime number.

We only need to test 2, 3, 5 and 7 as **dzielniki** factors , because $11 \times 11 = 121$ is bigger than 97.

98

98

Factors of 98: 1, 2, 7, 14, 49, 98

dzielniki

liczba pierwsza

98 has six factors , not two, so 98 is not a prime number.

99

99

Factors of 99: 1, 3, 9, 11, 33, 99

dzielniki

liczba pierwsza

99 has six factors , not two, so 99 is not a prime number.

100

100

Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

100 has nine dzielniki factors , not two, so 100 is not a liczba pierwsza prime number.

Prime factor decomposition

Rozkład na czynniki pierwsze

True or false?

$12 = 2^2 \times 3$ is the rozkład na czynniki pierwsze prime factor decomposition of 12

True or false?

$12 = 2^2 \times 3$ is the rozkład na czynniki pierwsze prime factor decomposition of 12

True. 2 and 3 are both liczba pierwsza prime, and $4 \times 3 = 12$.

True or false?

$2 \times 3 \times 4$ is the rozkład na czynniki pierwsze
prime factor decomposition of 24

True or false?

$2 \times 3 \times 4$ is the ^{rozkład na czynniki pierwsze} prime factor decomposition of 24

False. 4 is not ^{liczba pierwsza} prime.

$$24 = 2^3 \times 3$$

True or false?

Every whole number greater than 1 can be written as a

iloczyn liczby pierwsze
product of primes

True or false?

Every whole number greater than 1 can be written as a

iloczyn liczby pierwsze
product of primes

True. A liczba pierwsza
prime is its own decomposition, and any other
number keeps splitting until only liczby pierwsze
primes are left.

True or false?

A number can have two different ^{rozkłady na czynniki pierwsze} prime factor decompositions

True or false?

A number can have two different ^{rozkłady na czynniki pierwsze} prime factor decompositions

False. Apart from the order you write them in, every number has exactly one decomposition.

True or false?

$$2^3 \times 5 = 30$$

True or false?

$$2^3 \times 5 = 30$$

False. $2^3 \times 5 = 8 \times 5 = 40.$

$$30 = 2 \times 3 \times 5$$

Write 12 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 12 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

Write 18 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 18 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

Write 60 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 60 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

Write 84 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 84 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

Write 200 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 200 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$$

Write 360 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 360 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$360 = 2^3 \times 3^2 \times 5$$

Write 1001 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

Write 1001 as a ^{iloczyn}product of its ^{czynniki pierwsze}prime factors

$$1001 = 7 \times 143 = 7 \times 11 \times 13$$

Work out $2^3 \times 3 \times 5^2$ as an ordinary number

Work out $2^3 \times 3 \times 5^2$ as an ordinary number

$$8 \times 3 \times 25 = 600$$

$$n = 2^3 \times 3 \times 5.$$

Write $2n$ as a iloczyn product of czynniki pierwsze prime factors .

$$n = 2^3 \times 3 \times 5.$$

Write $2n$ as a iloczyn czynnik pierwsze product of prime factors .

Doubling adds one more dzielnik factor of 2:

$$2n = 2^4 \times 3 \times 5$$

$$720 = 2^4 \times 3^2 \times 5.$$

How many ^{dzielniki} factors does 720 have?

$$720 = 2^4 \times 3^2 \times 5.$$

How many ^{dzielniki} factors does 720 have?

Each ^{dzielnik} factor uses 0 to 4 twos, 0 to 2 threes and 0 or 1 five:

$$5 \times 3 \times 2 = 30 \text{ ^{dzielniki} factors}$$

największy wspólny dzielnik

HCF

najmniejsza wspólna wielokrotność

LCM

**and
from lists**

Find the HCF and LCM of 4 and 6.

Find the HCF and LCM of 4 and 6.

Factors of 4:

1, 2, 4

Factors of 6:

1, 2, 3, 6

Find the HCF and LCM of 4 and 6.

Factors of 4:

1, 2, 4

Factors of 6:

1, 2, 3, 6

Multiples of 4:

4, 8, 12, 16, 20,

24, 28, 32, 36, 40

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Find the HCF and LCM of 4 and 6.

Factors of 4:

1, ②, 4

Factors of 6:

1, ②, 3, 6

Multiples of 4:

4, 8, 12, 16, 20,

24, 28, 32, 36, 40

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Find the HCF and LCM of 4 and 6.

Factors of 4:

1, ②, 4

Factors of 6:

1, ②, 3, 6

Multiples of 4:

4, 8, ①②, 16, 20,

24, 28, 32, 36, 40

Multiples of 6:

6, ①②, 18, 24, 30,

36, 42, 48, 54, 60

Find the HCF and LCM of 6 and 8.

Find the HCF and LCM of 6 and 8.

Factors of 6:

1, 2, 3, 6

Factors of 8:

1, 2, 4, 8

Find the HCF and LCM of 6 and 8.

Factors of 6:

1, 2, 3, 6

Factors of 8:

1, 2, 4, 8

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, 72, 80

Find the HCF and LCM of 6 and 8.

Factors of 6:

1, ②, 3, 6

Factors of 8:

1, ②, 4, 8

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, 72, 80

Find the HCF and LCM of 6 and 8.

Factors of 6:

1, ②, 3, 6

Factors of 8:

1, ②, 4, 8

Multiples of 6:

6, 12, 18, ②4, 30,
36, 42, 48, 54, 60

Multiples of 8:

8, 16, ②4, 32, 40,
48, 56, 64, 72, 80

Find the HCF and LCM of 6 and 9.

Find the HCF and LCM of 6 and 9.

Factors of 6:

1, 2, 3, 6

Factors of 9:

1, 3, 9

Find the HCF and LCM of 6 and 9.

Factors of 6:

1, 2, 3, 6

Factors of 9:

1, 3, 9

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, 72, 81, 90

Find the HCF and LCM of 6 and 9.

Factors of 6:

1, 2, ③, 6

Factors of 9:

1, ③, 9

Multiples of 6:

6, 12, 18, 24, 30,

36, 42, 48, 54, 60

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, 72, 81, 90

Find the HCF and LCM of 6 and 9.

Factors of 6:

1, 2, ③, 6

Factors of 9:

1, ③, 9

Multiples of 6:

6, 12, ①8, 24, 30,

36, 42, 48, 54, 60

Multiples of 9:

9, ①8, 27, 36, 45,

54, 63, 72, 81, 90

Find the HCF and LCM of 8 and 12.

Find the HCF and LCM of 8 and 12.

Factors of 8:

1, 2, 4, 8

Factors of 12:

1, 2, 3, 4, 6, 12

Find the HCF and LCM of 8 and 12.

Factors of 8:

1, 2, 4, 8

Factors of 12:

1, 2, 3, 4, 6, 12

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, 72, 80

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 8 and 12.

Factors of 8:

1, 2, ④, 8

Factors of 12:

1, 2, 3, ④, 6, 12

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, 72, 80

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 8 and 12.

Factors of 8:

1, 2, ④, 8

Factors of 12:

1, 2, 3, ④, 6, 12

Multiples of 8:

8, 16, ②④, 32, 40,

48, 56, 64, 72, 80

Multiples of 12:

12, ②④, 36, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 10 and 15.

Find the HCF and LCM of 10 and 15.

Factors of 10:

1, 2, 5, 10

Factors of 15:

1, 3, 5, 15

Find the HCF and LCM of 10 and 15.

Factors of 10:

1, 2, 5, 10

Factors of 15:

1, 3, 5, 15

Multiples of 10:

10, 20, 30, 40, 50,

60, 70, 80, 90, 100

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Find the HCF and LCM of 10 and 15.

Factors of 10:

1, 2, 5, 10

Factors of 15:

1, 3, 5, 15

Multiples of 10:

10, 20, 30, 40, 50,

60, 70, 80, 90, 100

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Find the HCF and LCM of 10 and 15.

Factors of 10:

1, 2, 5, 10

Factors of 15:

1, 3, 5, 15

Multiples of 10:

10, 20, 30, 40, 50,

60, 70, 80, 90, 100

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Find the HCF and LCM of 9 and 12.

Find the HCF and LCM of 9 and 12.

Factors of 9:

1, 3, 9

Factors of 12:

1, 2, 3, 4, 6, 12

Find the HCF and LCM of 9 and 12.

Factors of 9:

1, 3, 9

Factors of 12:

1, 2, 3, 4, 6, 12

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, 72, 81, 90

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 9 and 12.

Factors of 9:

1, ③, 9

Factors of 12:

1, 2, ③, 4, 6, 12

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, 72, 81, 90

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 9 and 12.

Factors of 9:

1, ③, 9

Factors of 12:

1, 2, ③, 4, 6, 12

Multiples of 9:

9, 18, 27, ③⑥, 45,

54, 63, 72, 81, 90

Multiples of 12:

12, 24, ③⑥, 48, 60,

72, 84, 96, 108, 120

Find the HCF and LCM of 12 and 18.

Find the HCF and LCM of 12 and 18.

Factors of 12:

1, 2, 3, 4, 6, 12

Factors of 18:

1, 2, 3, 6, 9, 18

Find the HCF and LCM of 12 and 18.

Factors of 12:

1, 2, 3, 4, 6, 12

Factors of 18:

1, 2, 3, 6, 9, 18

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Multiples of 18:

18, 36, 54, 72, 90,

108, 126, 144, 162, 180

Find the HCF and LCM of 12 and 18.

Factors of 12:

1, 2, 3, 4, ⑥, 12

Factors of 18:

1, 2, 3, ⑥, 9, 18

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Multiples of 18:

18, 36, 54, 72, 90,

108, 126, 144, 162, 180

Find the HCF and LCM of 12 and 18.

Factors of 12:

1, 2, 3, 4, 6, 12

Factors of 18:

1, 2, 3, 6, 9, 18

Multiples of 12:

12, 24, 36, 48, 60,

72, 84, 96, 108, 120

Multiples of 18:

18, 36, 54, 72, 90,

108, 126, 144, 162, 180

Find the HCF and LCM of 14 and 21.

Find the HCF and LCM of 14 and 21.

Factors of 14:

1, 2, 7, 14

Factors of 21:

1, 3, 7, 21

Find the HCF and LCM of 14 and 21.

Factors of 14:

1, 2, 7, 14

Factors of 21:

1, 3, 7, 21

Multiples of 14:

14, 28, 42, 56, 70,

84, 98, 112, 126, 140

Multiples of 21:

21, 42, 63, 84, 105,

126, 147, 168, 189, 210

Find the HCF and LCM of 14 and 21.

Factors of 14:

1, 2, 7, 14

Factors of 21:

1, 3, 7, 21

Multiples of 14:

14, 28, 42, 56, 70,

84, 98, 112, 126, 140

Multiples of 21:

21, 42, 63, 84, 105,

126, 147, 168, 189, 210

Find the HCF and LCM of 14 and 21.

Factors of 14:

1, 2, 7, 14

Factors of 21:

1, 3, 7, 21

Multiples of 14:

14, 28, 42, 56, 70,

84, 98, 112, 126, 140

Multiples of 21:

21, 42, 63, 84, 105,

126, 147, 168, 189, 210

Find the HCF and LCM of 15 and 20.

Find the HCF and LCM of 15 and 20.

Factors of 15:

1, 3, 5, 15

Factors of 20:

1, 2, 4, 5, 10, 20

Find the HCF and LCM of 15 and 20.

Factors of 15:

1, 3, 5, 15

Factors of 20:

1, 2, 4, 5, 10, 20

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Multiples of 20:

20, 40, 60, 80, 100,

120, 140, 160, 180, 200

Find the HCF and LCM of 15 and 20.

Factors of 15:

1, 3, 5, 15

Factors of 20:

1, 2, 4, 5, 10, 20

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Multiples of 20:

20, 40, 60, 80, 100,

120, 140, 160, 180, 200

Find the HCF and LCM of 15 and 20.

Factors of 15:

1, 3, 5, 15

Factors of 20:

1, 2, 4, 5, 10, 20

Multiples of 15:

15, 30, 45, 60, 75,

90, 105, 120, 135, 150

Multiples of 20:

20, 40, 60, 80, 100,

120, 140, 160, 180, 200

Find the HCF and LCM of 8 and 9.

Find the HCF and LCM of 8 and 9.

Factors of 8:

1, 2, 4, 8

Factors of 9:

1, 3, 9

Find the HCF and LCM of 8 and 9.

Factors of 8:

1, 2, 4, 8

Factors of 9:

1, 3, 9

Multiples of 8:

8, 16, 24, 32, 40,
48, 56, 64, 72, 80

Multiples of 9:

9, 18, 27, 36, 45,
54, 63, 72, 81, 90

Find the HCF and LCM of 8 and 9.

Factors of 8:

①, 2, 4, 8

Factors of 9:

①, 3, 9

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, 72, 80

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, 72, 81, 90

Find the HCF and LCM of 8 and 9.

Factors of 8:

①, 2, 4, 8

Factors of 9:

①, 3, 9

Multiples of 8:

8, 16, 24, 32, 40,

48, 56, 64, ⑦2 80

Multiples of 9:

9, 18, 27, 36, 45,

54, 63, ⑦2 81, 90

największy wspólny dzielnik and najmniejsza wspólna wielokrotność

HCF **LCM**

by rozkład na czynniki pierwsze **prime factor decomposition**

True or false?

The ^{największy wspólny dzielnik}
HCF of two numbers is always smaller
than both of them

True or false?

The ^{największy wspólny dzielnik}
HCF of two numbers is always smaller
than both of them

False. The ^{największy wspólny dzielnik}
HCF of 6 and 12 is 6, which is
one of the numbers itself.

True or false?

The ^{najmniejsza wspólna wielokrotność}
LCM of two numbers is always a
^{wielokrotność}
multiple of both of them

True or false?

The ^{najmniejsza wspólna wielokrotność}
LCM of two numbers is always a
^{wielokrotność}
multiple of both of them

True. That is exactly what a common ^{wielokrotność}
multiple is; the
^{najmniejsza wspólna wielokrotność}
LCM is the smallest one.

True or false?

If $a = 2^2 \times 3$ and $b = 2 \times 3^2$,

then the największy wspólny dzielnik
HCF of a and b is 2×3

True or false?

If $a = 2^2 \times 3$ and $b = 2 \times 3^2$,

then the największy wspólny dzielnik
HCF of a and b is 2×3

True. Take the *lowest* potęga **power** of each liczba pierwsza **prime** that

appears in both: $2^1 \times 3^1 = 6$.

True or false?

HCF $\times$ LCM gives the ^{iloczyn}product of the two numbers

True or false?

HCF $\times$ LCM gives the ^{iloczyn}product of the two numbers

True. Every ^{liczba pierwsza}prime is counted once at its lowest ^{potęga}power

and once at its highest, which is the same as counting both
numbers.

True or false?

The ^{najmniejsza wspólna wielokrotność}
lowest common multiple of 6 and 8 is 48

True or false?

The najmniejsza wspólna wielokrotność
lowest common multiple of 6 and 8 is 48

False. $6 = 2 \times 3$ and $8 = 2^3$, so the

najmniejsza wspólna wielokrotność
LCM is $2^3 \times 3 = 24$.

$$12 = 2^2 \times 3 \quad \text{and} \quad 18 = 2 \times 3^2$$

Find the największy wspólny dzielnik
HCF of 12 and 18.

$$12 = 2^2 \times 3 \quad \text{and} \quad 18 = 2 \times 3^2$$

Find the ^{największy wspólny dzielnik}
HCF of 12 and 18.

^{potęga} **Lowest** ^{liczba pierwsza}
power of each shared prime :

$$\text{HCF} = 2 \times 3 = 6$$

$$12 = 2^2 \times 3 \quad \text{and} \quad 18 = 2 \times 3^2$$

Find the najmniejsza wspólna wielokrotność
LCM of 12 and 18.

$$12 = 2^2 \times 3 \quad \text{and} \quad 18 = 2 \times 3^2$$

Find the najmniejsza wspólna wielokrotność
LCM of 12 and 18.

Highest potęga power of each liczba pierwsza prime :

$$\text{LCM} = 2^2 \times 3^2 = 36$$

$$24 = 2^3 \times 3 \quad \text{and} \quad 60 = 2^2 \times 3 \times 5$$

Find the największy wspólny dzielnik
HCF of 24 and 60.

$$24 = 2^3 \times 3 \quad \text{and} \quad 60 = 2^2 \times 3 \times 5$$

Find the największy wspólny dzielnik
HCF of 24 and 60.

$$\text{HCF} = 2^2 \times 3 = 12$$

$$24 = 2^3 \times 3 \quad \text{and} \quad 60 = 2^2 \times 3 \times 5$$

Find the najmniejsza wspólna wielokrotność
LCM of 24 and 60.

$$24 = 2^3 \times 3 \quad \text{and} \quad 60 = 2^2 \times 3 \times 5$$

Find the najmniejsza wspólna wielokrotność
LCM of 24 and 60.

$$\text{LCM} = 2^3 \times 3 \times 5 = 120$$

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM of 84 and 120

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM of 84 and 120

$$84 = 2^2 \times 3 \times 7 \quad 120 = 2^3 \times 3 \times 5$$

$$\text{HCF} = 2^2 \times 3 = 12$$

$$\text{LCM} = 2^3 \times 3 \times 5 \times 7 = 840$$

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM of 126 and 180

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM of 126 and 180

$$126 = 2 \times 3^2 \times 7 \qquad 180 = 2^2 \times 3^2 \times 5$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^2 \times 3^2 \times 5 \times 7 = 1260$$

$$a = 2^3 \times 3^2 \times 5 \quad \text{and} \quad b = 2^2 \times 3 \times 5^2$$

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM, in postać potęgowa
index form.

$$a = 2^3 \times 3^2 \times 5 \quad \text{and} \quad b = 2^2 \times 3 \times 5^2$$

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM, in postać potęgowa
index form.

$$\text{HCF} = 2^2 \times 3 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5^2$$

Two numbers have $\text{największy wspólny dzielnik}$
 HCF 6 and
 $\text{najmniejsza wspólna wielokrotność}$
 LCM 72.

One of the numbers is 24. Find the other.

Two numbers have ^{największy wspólny dzielnik}
HCF 6 and
^{najmniejsza wspólna wielokrotność}
LCM 72.

One of the numbers is 24. Find the other.

$\text{HCF} \times \text{LCM} = 6 \times 72 = 432$, which is the ^{iloczyn}
product of the two numbers.

$$432 \div 24 = 18$$

Find the największy wspólny dzielnik
HCF and the

najmniejsza wspólna wielokrotność
LCM of

$$2^5 \times 3^2 \times 11 \quad \text{and} \quad 2^3 \times 3^4 \times 7$$

Leave your answers in postać potęgowa
index form .

Find the największy wspólny dzielnik
HCF and the
najmniejsza wspólna wielokrotność
LCM of

$$2^5 \times 3^2 \times 11 \quad \text{and} \quad 2^3 \times 3^4 \times 7$$

Leave your answers in postać potęgowa
index form .

$$\text{HCF} = 2^3 \times 3^2$$

$$\text{LCM} = 2^5 \times 3^4 \times 7 \times 11$$

Two numbers have $\text{największy wspólny dzielnik}$
 HCF 12 and
 $\text{najmniejsza wspólna wielokrotność}$
 LCM 180.

Find the numbers. Is there more than one answer?

Two numbers have $\text{największy wspólny dzielnik}$
 HCF 12 and
 $\text{najmniejsza wspólna wielokrotność}$
 LCM 180.

Find the numbers. Is there more than one answer?

iloczyn
Their product is $12 \times 180 = 2160$, and both are wielokrotności
 multiples of 12.

12 and 180 or 36 and 60

Both pairs work, so there is more than one answer.

Square numbers from prime factors

Liczby kwadratowe
czynniki pierwsze

True or false?

A number is square exactly when every ^{potęga}power in its
^{rozkład na czynniki pierwsze}prime factor decomposition is even

True or false?

A number is square exactly when every ^{potęga}power in its
^{rozkład na czynniki pierwsze}prime factor decomposition is even

True. Halving every even ^{potęga}power gives the ^{pierwiastek}square root.

True or false?

$2^3 \times 3^2$ is a liczba kwadratowa
square number

True or false?

$2^3 \times 3^2$ is a liczba kwadratowa
square number

False. The potęga
power of 2 is odd, so the number cannot be split
into two equal halves.

True or false?

$2^4 \times 5^6$ is a liczba kwadratowa
square number

True or false?

$2^4 \times 5^6$ is a liczba kwadratowa
square number

True. $2^4 \times 5^6 = (2^2 \times 5^3)^2$

True or false?

Every ^{liczba kwadratowa} square number has an even number of ^{dzielniki} factors

True or false?

Every ^{liczba kwadratowa} **square number** has an even number of ^{dzielniki} **factors**

False. ^{Liczby kwadratowe} **Square numbers** have an *odd* number of ^{dzielniki} **factors**:

16 has 1, 2, 4, 8, 16.

True or false?

If n is a liczba kwadratowa square number, then $4n$ is a liczba kwadratowa square number too

True or false?

If n is a liczba kwadratowa **square number**, then $4n$ is a liczba kwadratowa **square number** too

True. Multiplying by $4 = 2^2$ keeps every potęga **power** even.

Use ^{czynniki pierwsze} prime factors to show that 36 is a ^{liczba kwadratowa} square number

Use ^{czynniki pierwsze} prime factors to show that 36 is a ^{liczba kwadratowa} square number

$$36 = 2^2 \times 3^2$$

Both ^{potęgi} powers are even, and $36 = (2 \times 3)^2 = 6^2$.

Is $2^2 \times 3^2 \times 5^2$ a liczba kwadratowa
square number?

If so, what is its pierwiastek
square root?

Is $2^2 \times 3^2 \times 5^2$ a ^{liczba kwadratowa} **square number**?

If so, what is its ^{pierwiastek} **square root**?

Yes — every ^{potęga} **power** is even.

$$\sqrt{2^2 \times 3^2 \times 5^2} = 2 \times 3 \times 5 = 30$$

Is $2^3 \times 3^2$ a liczba kwadratowa
square number?

Is $2^3 \times 3^2$ a ^{liczba kwadratowa} square number?

No. Halving the ^{potęgi} powers would need $2^{1.5}$,

so the ^{potęga} power of 2 must be even and it is not.

Work out $\sqrt{2^6 \times 3^4}$

Work out $\sqrt{2^6 \times 3^4}$

Halve each ^{potęga} power:

$$2^3 \times 3^2 = 8 \times 9 = 72$$

Use ^{czynniki pierwsze} prime factors to decide whether 400 is a ^{liczba kwadratowa} square number

Use ^{czynniki pierwsze} prime factors to decide whether 400 is a ^{liczba kwadratowa} square number

$$400 = 2^4 \times 5^2$$

Both ^{potęgi} powers are even, so $400 = (2^2 \times 5)^2 = 20^2$.

Use ^{czynniki pierwsze} prime factors to decide whether 150 is a ^{liczba kwadratowa} square number

Use ^{czynniki pierwsze} prime factors to decide whether 150 is a ^{liczba kwadratowa} square number

$$150 = 2 \times 3 \times 5^2$$

^{potęgi} The powers of 2 and 3 are odd, so 150 is not square.

Work out $\sqrt{2^8 \times 3^2 \times 5^4}$

Work out $\sqrt{2^8 \times 3^2 \times 5^4}$

Halve each ^{potęga} power:

$$2^4 \times 3 \times 5^2 = 16 \times 3 \times 25 = 1200$$

$$n = 2^3 \times 3 \times 5^2$$

Find the smallest whole number k so that nk is a

liczba kwadratowa
square number.

$$n = 2^3 \times 3 \times 5^2$$

Find the smallest whole number k so that nk is a

liczba kwadratowa
square number.

potęgi

The powers of 2 and 3 are odd, so each needs one more:

$$k = 2 \times 3 = 6, \text{ giving } nk = 2^4 \times 3^2 \times 5^2.$$

$$504 = 2^3 \times 3^2 \times 7$$

Find the smallest number 504 can be multiplied by to give a

liczba kwadratowa
square number.

$$504 = 2^3 \times 3^2 \times 7$$

Find the smallest number 504 can be multiplied by to give a

liczba kwadratowa
square number.

potęgi
The powers of 2 and 7 are odd:

multiply by $2 \times 7 = 14$.

$$504 \times 14 = 7056 = 84^2$$

$$1080 = 2^3 \times 3^3 \times 5$$

Find the smallest number 1080 can be divided by to give a

liczba kwadratowa
square number.

$$1080 = 2^3 \times 3^3 \times 5$$

Find the smallest number 1080 can be divided by to give a

liczba kwadratowa
square number.

Remove one 2, one 3 and the 5:

divide by $2 \times 3 \times 5 = 30$.

$$1080 \div 30 = 36 = 6^2$$

Problem solving with **HCF** and **LCM**

największy wspólny dzielnik
najmniejsza wspólna wielokrotność

True or false?

To find when two repeating events next happen together, you

use the najmniejsza wspólna wielokrotność
LCM

True or false?

To find when two repeating events next happen together, you

use the najmniejsza wspólna wielokrotność
LCM

True. You are looking for the first time that is a wielokrotność
multiple
of both gaps.

True or false?

To split two collections into the largest identical bags, you use

the najmniejsza wspólna wielokrotność
LCM

True or false?

To split two collections into the largest identical bags, you use

the najmniejsza wspólna wielokrotność
LCM

False. Use the największy wspólny dzielnik
HCF — the bag size has to
dzielnik
be a factor of both amounts.

True or false?

Cutting two ropes into equal pieces, as long as possible with none left over, uses the największy wspólny dzielnik
HCF

True or false?

Cutting two ropes into equal pieces, as long as possible with none left over, uses the ^{największy wspólny dzielnik} **HCF**

True. The piece length must be a ^{dzielnik} **factor** of both rope lengths.

True or false?

The najmniejsza wspólna wielokrotność
LCM of two numbers is never
smaller than either of them

True or false?

The ^{najmniejsza wspólna wielokrotność}
LCM of two numbers is never
smaller than either of them

True. It is a ^{wielokrotność}
multiple of each number, so it is at least as
big as both.

True or false?

Two events repeat every 4 minutes and every 6 minutes,
so they coincide every 24 minutes

True or false?

Two events repeat every 4 minutes and every 6 minutes,
so they coincide every 24 minutes

False. They coincide every $\text{LCM}(4, 6) = 12$ minutes.

Two bells ring together now.
One rings every 4 minutes, the other every 6 minutes.
When do they next ring together?

Two bells ring together now.

One rings every 4 minutes, the other every 6 minutes.

When do they next ring together?

Common ^{wielokrotność} multiple , so use the

^{najmniejsza wspólna wielokrotność}
LCM :

$\text{LCM}(4, 6) = 12$ — in 12 minutes.

Two ribbons, 18 cm and 24 cm long, are cut into equal pieces,
as long as possible with none left over.

How long is each piece?

Two ribbons, 18 cm and 24 cm long, are cut into equal pieces,
as long as possible with none left over.

How long is each piece?

The length must be a ^{dzielnik} factor of both, so use the

^{największy wspólny dzielnik}
HCF :

$\text{HCF}(18, 24) = 6$ — each piece is 6 cm.

36 pens and 48 pencils are shared into identical packs with nothing left over.

What is the greatest number of packs?

36 pens and 48 pencils are shared into identical packs with nothing left over.

What is the greatest number of packs?

$$36 = 2^2 \times 3^2 \qquad 48 = 2^4 \times 3$$

$$\text{HCF} = 2^2 \times 3 = 12 \text{ packs,}$$

each with 3 pens and 4 pencils.

Two lighthouses flash together at 8pm.
One flashes every 15 seconds, the other every 18 seconds.
When do they next flash together?

Two lighthouses flash together at 8pm.
One flashes every 15 seconds, the other every 18 seconds.
When do they next flash together?

$$\text{LCM}(15, 18) = 90 \text{ seconds}$$

At 8:01:30pm.

Rectangular tiles measure 12 cm by 18 cm.
What is the smallest square they can tile exactly?

Rectangular tiles measure 12 cm by 18 cm.

What is the smallest square they can tile exactly?

The side must be a ^{wielokrotność} multiple of both, so use the

^{najmniejsza wspólna wielokrotność}
LCM :

$$\text{LCM}(12, 18) = 36$$

A 36 cm by 36 cm square.

A bus leaves every 24 minutes and a tram every 36 minutes.
Both leave at 9:00. When do they next leave together?

A bus leaves every 24 minutes and a tram every 36 minutes.
Both leave at 9:00. When do they next leave together?

$$\text{LCM}(24, 36) = 72 \text{ minutes}$$

At 10:12.

Two cogs, with 18 and 24 teeth, start meshed at a marked tooth.

How many turns does each make before the marks meet again?

Two cogs, with 18 and 24 teeth, start meshed at a marked tooth.

How many turns does each make before the marks meet again?

$\text{LCM}(18, 24) = 72$ teeth must pass.

$72 \div 18 = 4$ turns of the small cog,

$72 \div 24 = 3$ turns of the large cog.

A floor measures 360 cm by 504 cm.
It is tiled with identical square tiles, as large as possible.
Find the tile size and the number of tiles.

A floor measures 360 cm by 504 cm.

It is tiled with identical square tiles, as large as possible.

Find the tile size and the number of tiles.

$$360 = 2^3 \times 3^2 \times 5 \qquad 504 = 2^3 \times 3^2 \times 7$$

$\text{HCF} = 2^3 \times 3^2 = 72$, so the tiles are 72 cm square.

$$5 \times 7 = 35 \text{ tiles.}$$

Three runners take 60 s, 72 s and 90 s per lap.
They start together. When are they next all at the start line?

Three runners take 60 s, 72 s and 90 s per lap.

They start together. When are they next all at the start line?

$$60 = 2^2 \times 3 \times 5 \quad 72 = 2^3 \times 3^2 \quad 90 = 2 \times 3^2 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360 \text{ s}$$

After 6 minutes.

Hot dogs come in packs of 10, buns in packs of 8 and sauce sachets in packs of 12.

What is the smallest number of complete hot dogs Sam can make with nothing left over, and how many packs of each is that?

Hot dogs come in packs of 10, buns in packs of 8 and sauce sachets in packs of 12.

What is the smallest number of complete hot dogs Sam can make with nothing left over, and how many packs of each is that?

$$\text{LCM}(10, 8, 12) = 2^3 \times 3 \times 5 = 120$$

120 hot dogs: 12 packs of hot dogs,
15 packs of buns and 10 packs of sachets.