

Factorising and Algebraic Fractions: Quick Questions

Mini whiteboard questions, with solutions

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Factorising into single brackets

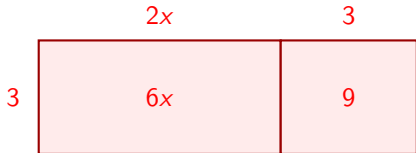
True or false?

$$6x + 9 = 3(2x + 3)$$

True or false?

$$6x + 9 = 3(2x + 3)$$

True. The highest common factor of $6x$ and 9 is 3 , and $3 \times 2x = 6x$,
 $3 \times 3 = 9$.



True or false?

$8x + 12$ is fully factorised as $2(4x + 6)$

True or false?

$8x + 12$ is fully factorised as $2(4x + 6)$

False. 2 is a common factor of $8x$ and 12, but it is not the highest common factor, so the expression is not fully factorised.

The highest common factor of $8x$ and 12 is 4, so $8x + 12 = 4(2x + 3)$.

True or false?

$$5x + 10y = 5(x + 10y)$$

True or false?

$$5x + 10y = 5(x + 10y)$$

False. Every term inside the bracket must be divided by the common factor, not only the first term.

$$10y \div 5 = 2y, \text{ so } 5x + 10y = 5(x + 2y).$$

True or false?

$$x^2 + x = x(x)$$

True or false?

$$x^2 + x = x(x)$$

False. $x(x)$ is x^2 , so the second term has been lost.

$$x \div x = 1, \text{ so } x^2 + x = x(x + 1).$$

True or false?

$$3x + 3 = 3(x + 1)$$

True or false?

$$3x + 3 = 3(x + 1)$$

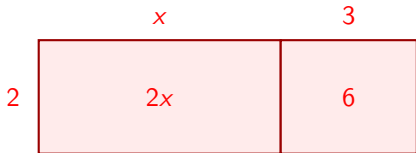
True. The second term is entirely the common factor, and $3 \div 3 = 1$, so a 1 is left inside the bracket.

Factorise $2x + 6$

Factorise $2x + 6$

The highest common factor of $2x$ and 6 is 2 .

$$2x + 6 = 2(x + 3)$$



Factorise $9x + 24$

Factorise $9x + 24$

The highest common factor of $9x$ and 24 is 3 .

$$9x \div 3 = 3x \text{ and } 24 \div 3 = 8.$$

$$9x + 24 = 3(3x + 8)$$

Factorise $6y - 15$

Factorise $6y - 15$

The highest common factor of $6y$ and 15 is 3 .

$$6y - 15 = 3(2y - 5)$$

Factorise $10a + 15b$

Factorise $10a + 15b$

The highest common factor of $10a$ and $15b$ is 5.

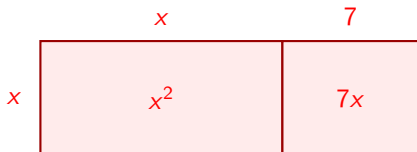
$$10a + 15b = 5(2a + 3b)$$

Factorise $x^2 + 7x$

Factorise $x^2 + 7x$

The highest common factor of x^2 and $7x$ is x .

$$x^2 + 7x = x(x + 7)$$

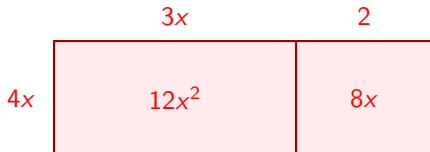


Factorise $12x^2 + 8x$

Factorise $12x^2 + 8x$

The highest common factor of $12x^2$ and $8x$ is $4x$.

$$12x^2 + 8x = 4x(3x + 2)$$



Factorise $6x^3 - 9x^2$

Factorise $6x^3 - 9x^2$

The highest common factor of the numbers is 3 and of the powers of x is x^2 , so the highest common factor is $3x^2$.

$$6x^3 - 9x^2 = 3x^2(2x - 3)$$

Factorise $4x^2y + 6xy^2$

Factorise $4x^2y + 6xy^2$

Both terms contain 2, x and y , so the highest common factor is $2xy$.

$$4x^2y + 6xy^2 = 2xy(2x + 3y)$$

Factorise $8x^3 + 12x^2 + 20x$

Factorise $8x^3 + 12x^2 + 20x$

The highest common factor of $8x^3$, $12x^2$ and $20x$ is $4x$.

$$8x^3 + 12x^2 + 20x = 4x(2x^2 + 3x + 5)$$

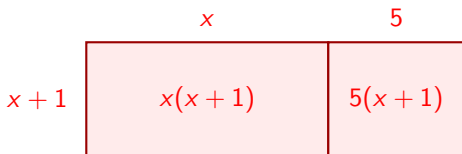
	$2x^2$	$3x$	5
$4x$	$8x^3$	$12x^2$	$20x$

Factorise $5(x + 1) + x(x + 1)$

Factorise $5(x + 1) + x(x + 1)$

The bracket $(x + 1)$ is a common factor of both terms.

$$5(x + 1) + x(x + 1) = (x + 1)(x + 5)$$



Factorising into double brackets

True or false?

$$x^2 - 9 = (x - 3)(x - 3)$$

True or false?

$$x^2 - 9 = (x - 3)(x - 3)$$

False. $(x - 3)(x - 3)$ expands to $x^2 - 6x + 9$.

The signs must be opposite: $x^2 - 9 = (x + 3)(x - 3)$.

	x	3
x	x^2	$3x$
-3	$-3x$	-9

True or false?

$$x^2 + 9 = (x + 3)(x + 3)$$

True or false?

$$x^2 + 9 = (x + 3)(x + 3)$$

False. $(x + 3)(x + 3)$ expands to $x^2 + 6x + 9$.

$x^2 + 9$ is a sum of two squares, and it does not factorise.

True or false?

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

True or false?

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

True. $5 \times (-3) = -15$ and $5 + (-3) = 2$.

True or false?

$$x^2 - 2x - 8 = (x + 4)(x - 2)$$

True or false?

$$x^2 - 2x - 8 = (x + 4)(x - 2)$$

False. $(x + 4)(x - 2)$ expands to $x^2 + 2x - 8$, so the signs are the wrong way round.

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

True or false?

To factorise $x^2 + 7x + 12$ we need two numbers
that multiply to 12 and add to 7

True or false?

To factorise $x^2 + 7x + 12$ we need two numbers that multiply to 12 and add to 7

True. The two numbers multiply to the constant term and add to the coefficient of x .

The numbers are 3 and 4, so $x^2 + 7x + 12 = (x + 3)(x + 4)$.

Factorise $x^2 + 5x + 6$

Factorise $x^2 + 5x + 6$

Two numbers that multiply to 6 and add to 5: 2 and 3.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

	x	2
x	x^2	$2x$
3	$3x$	6

Factorise $x^2 + 9x + 20$

Factorise $x^2 + 9x + 20$

Two numbers that multiply to 20 and add to 9: 4 and 5.

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

Factorise $x^2 + 3x - 10$

Factorise $x^2 + 3x - 10$

Two numbers that multiply to -10 and add to 3 : 5 and -2 .

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

	x	5
x	x^2	$5x$
-2	$-2x$	-10

Factorise $x^2 - 7x + 12$

Factorise $x^2 - 7x + 12$

The constant term is positive and the coefficient of x is negative, so both numbers are negative: -3 and -4 .

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Factorise $x^2 - 2x - 15$

Factorise $x^2 - 2x - 15$

Two numbers that multiply to -15 and add to -2 :
 -5 and 3 .

$$x^2 - 2x - 15 = (x - 5)(x + 3)$$

Factorise $x^2 - 49$

Factorise $x^2 - 49$

This is a difference of two squares, with $x^2 = (x)^2$
and $49 = 7^2$.

$$x^2 - 49 = (x + 7)(x - 7)$$

Factorise $2x^2 + 7x + 3$

Factorise $2x^2 + 7x + 3$

The $2x^2$ comes from $2x \times x$, and the 3 from 1×3 .

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

	$2x$	1
x	$2x^2$	x
3	$6x$	3

Factorise $2x^2 + 10x + 12$

Factorise $2x^2 + 10x + 12$

Take the common factor of 2 out first.

$$2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$$

$$x^2 + 5x + 6 = (x + 2)(x + 3), \text{ so } 2x^2 + 10x + 12 = 2(x + 2)(x + 3).$$

	x^2	$5x$	6
2	$2x^2$	$10x$	12

Factorise $9x^2 - 16$

Factorise $9x^2 - 16$

This is a difference of two squares, with $9x^2 = (3x)^2$
and $16 = 4^2$.

$$9x^2 - 16 = (3x + 4)(3x - 4)$$

Factorise $6x^2 - x - 12$

Factorise $6x^2 - x - 12$

The $6x^2$ comes from $3x \times 2x$, and the -12 from $4 \times (-3)$.

$$6x^2 - x - 12 = (3x + 4)(2x - 3)$$

	$3x$	4
$2x$	$6x^2$	$8x$
-3	$-9x$	-12

Simplifying algebraic fractions with single brackets

True or false?

$$\frac{x + 2}{2} = x$$

True or false?

$$\frac{x + 2}{2} = x$$

False. The 2 on the bottom divides the whole numerator, not the term 2 on its own.

$x + 2$ has no common factor, so $\frac{x + 2}{2}$ does not simplify.

When $x = 4$: $\frac{4 + 2}{2} = 3$, not 4.

True or false?

$$\frac{3x + 6}{3} = x + 2$$

True or false?

$$\frac{3x + 6}{3} = x + 2$$

True. 3 is a factor of the whole numerator: $3x + 6 = 3(x + 2)$.

$$\frac{3(x + 2)}{3} = x + 2$$

True or false?

$$\frac{x+3}{x} = 3$$

True or false?

$$\frac{x + 3}{x} = 3$$

False. x is a term of the numerator, not a factor of the whole numerator.

$\frac{x + 3}{x}$ does not simplify.

When $x = 1$: $\frac{1 + 3}{1} = 4$, not 3.

True or false?

$$\frac{6x + 9}{3} = 2x + 9$$

True or false?

$$\frac{6x + 9}{3} = 2x + 9$$

False. Every term of the numerator is divided by 3, not the first term on its own.

$6x + 9 = 3(2x + 3)$, so the fraction simplifies to $2x + 3$.

True or false?

$$\frac{5x + 10}{5x} = \frac{x + 2}{x}$$

True or false?

$$\frac{5x + 10}{5x} = \frac{x + 2}{x}$$

True. 5 is a common factor of the whole numerator and the whole denominator.

$$\frac{\cancel{5}(x + 2)}{\cancel{5}x} = \frac{x + 2}{x}$$

Simplify $\frac{2x + 10}{2}$

Simplify $\frac{2x + 10}{2}$

Factorise the numerator.

$$\frac{2x + 10}{2} = \frac{\cancel{2}(x + 5)}{\cancel{2}} = x + 5$$

	x	5
2	$2x$	10

Simplify $\frac{5x - 15}{5}$

Simplify $\frac{5x - 15}{5}$

The highest common factor of the numerator is 5.

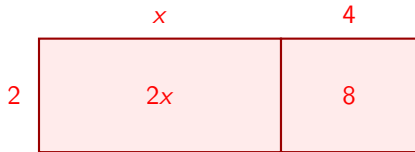
$$\frac{5(x - 3)}{5} = \frac{\cancel{5}(x - 3)}{\cancel{5}} = x - 3$$

Simplify $\frac{2x + 8}{x + 4}$

Simplify $\frac{2x + 8}{x + 4}$

Factorise the numerator.

$$\frac{2x + 8}{x + 4} = \frac{2(\cancel{x + 4})}{(\cancel{x + 4})} = 2$$



Simplify $\frac{3x - 12}{x - 4}$

Simplify $\frac{3x - 12}{x - 4}$

Factorise the numerator: $3x - 12 = 3(x - 4)$.

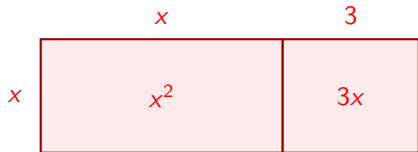
$$\frac{3(\cancel{x - 4})}{(\cancel{x - 4})} = 3$$

Simplify $\frac{x^2 + 3x}{x}$

Simplify $\frac{x^2 + 3x}{x}$

The highest common factor of the numerator is x .

$$\frac{x(x+3)}{x} = \frac{\cancel{x}(x+3)}{\cancel{x}} = x+3$$



Simplify $\frac{3x^2 + 6x + 9}{3}$

Simplify $\frac{3x^2 + 6x + 9}{3}$

3 is a factor of all three terms of the numerator.

$$\frac{\cancel{3}(x^2 + 2x + 3)}{\cancel{3}} = x^2 + 2x + 3$$

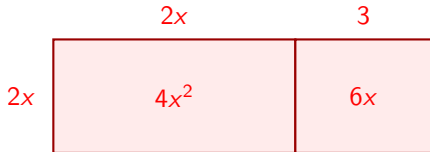
	x^2	$2x$	3
3	$3x^2$	$6x$	9

Simplify $\frac{4x^2 + 6x}{2x}$

Simplify $\frac{4x^2 + 6x}{2x}$

The highest common factor of the numerator is $2x$.

$$\frac{2x(2x + 3)}{2x} = \frac{\cancel{2x}(2x + 3)}{\cancel{2x}} = 2x + 3$$



Simplify $\frac{4x + 12}{6x + 18}$

Simplify $\frac{4x + 12}{6x + 18}$

Factorise the numerator and the denominator.

$$\begin{aligned}\frac{4x + 12}{6x + 18} &= \frac{4(x + 3)}{6(x + 3)} \\ \frac{\cancel{4(x + 3)}}{\cancel{6(x + 3)}} &= \frac{4}{6} = \frac{2}{3}\end{aligned}$$

Simplify $\frac{10}{5x + 20}$

Simplify $\frac{10}{5x + 20}$

Factorise the denominator: $5x + 20 = 5(x + 4)$.

$$\frac{\cancel{5} \times 2}{\cancel{5}(x + 4)} = \frac{2}{x + 4}$$

Simplify $\frac{6x^2 + 9x}{4x^2 + 6x}$

Simplify $\frac{6x^2 + 9x}{4x^2 + 6x}$

The highest common factor is $3x$ on the top and $2x$ on the bottom.

$$\begin{aligned}\frac{6x^2 + 9x}{4x^2 + 6x} &= \frac{3x(2x + 3)}{2x(2x + 3)} \\ \frac{\cancel{3x(2x + 3)}}{\cancel{2x(2x + 3)}} &= \frac{3}{2}\end{aligned}$$

Simplifying algebraic fractions with double brackets

True or false?

$$\frac{x^2 + 5x + 6}{x + 2} = x + 3$$

True or false?

$$\frac{x^2 + 5x + 6}{x + 2} = x + 3$$

True. $x^2 + 5x + 6 = (x + 2)(x + 3)$, so $(x + 2)$ is a factor of the whole numerator and cancels.

$$\frac{\cancel{(x + 2)}(x + 3)}{\cancel{(x + 2)}} = x + 3$$

True or false?

$$\frac{x^2 + 7x}{x^2 + 2x} = \frac{7}{2}$$

True or false?

$$\frac{x^2 + 7x}{x^2 + 2x} = \frac{7}{2}$$

False. x^2 is a term, not a factor of the whole numerator, so it cannot be cancelled.

$$\frac{x(x + 7)}{x(x + 2)} = \frac{x + 7}{x + 2}$$

True or false?

$$\frac{x - 3}{3 - x} = 1$$

True or false?

$$\frac{x-3}{3-x} = 1$$

False. The two brackets are not the same: $3-x = -(x-3)$.

$$\frac{x-3}{-(x-3)} = -1$$

When $x = 5$: $\frac{2}{-2} = -1$.

True or false?

$$\frac{x^2 - 25}{x - 5} = x + 5$$

True or false?

$$\frac{x^2 - 25}{x - 5} = x + 5$$

True. The numerator is a difference of two squares:

$$x^2 - 25 = (x - 5)(x + 5).$$

$$\frac{\cancel{(x - 5)}(x + 5)}{\cancel{(x - 5)}} = x + 5$$

True or false?

$$\frac{x^2 - 16}{x^2 + 8x + 16} = 1$$

True or false?

$$\frac{x^2 - 16}{x^2 + 8x + 16} = 1$$

False. $(x - 4)$ and $(x + 4)$ are different brackets and do not cancel.

$$\frac{(x - 4)(x + 4)}{(x + 4)(x + 4)} = \frac{x - 4}{x + 4}$$

Simplify $\frac{x^2 + 8x + 15}{x + 5}$

Simplify $\frac{x^2 + 8x + 15}{x + 5}$

Factorise the numerator. Two numbers with a sum of 8 and a product of 15: 3 and 5.

$$\frac{(x + 3)\cancel{(x + 5)}}{\cancel{(x + 5)}} = x + 3$$

Simplify $\frac{x^2 + 7x + 12}{x + 3}$

Simplify $\frac{x^2 + 7x + 12}{x + 3}$

Two numbers with a sum of 7 and a product of 12: 3 and 4.

$$\frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}} = x + 4$$

Simplify $\frac{x^2 - 36}{x + 6}$

Simplify $\frac{x^2 - 36}{x + 6}$

The numerator is a difference of two squares: $x^2 - 36 = (x + 6)(x - 6)$.

$$\frac{\cancel{(x + 6)}(x - 6)}{\cancel{(x + 6)}} = x - 6$$

Simplify $\frac{x^2 + 2x - 8}{x - 2}$

Simplify $\frac{x^2 + 2x - 8}{x - 2}$

Two numbers with a sum of 2 and a product of -8 : 4 and -2 .

$$\frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}} = x+4$$

	x	4
x	x^2	$4x$
-2	$-2x$	-8

Simplify $\frac{x^2 + 6x + 8}{x^2 + 5x + 4}$

Simplify $\frac{x^2 + 6x + 8}{x^2 + 5x + 4}$

Factorise the numerator and the denominator.

$$\begin{aligned}\frac{x^2 + 6x + 8}{x^2 + 5x + 4} &= \frac{(x + 2)(x + 4)}{(x + 1)(x + 4)} \\ \frac{(x + 2)\cancel{(x + 4)}}{(x + 1)\cancel{(x + 4)}} &= \frac{x + 2}{x + 1}\end{aligned}$$

Simplify $\frac{x^2 - x - 6}{x^2 - 9}$

Simplify $\frac{x^2 - x - 6}{x^2 - 9}$

The denominator is a difference of two squares.

$$\begin{aligned}\frac{x^2 - x - 6}{x^2 - 9} &= \frac{(x - 3)(x + 2)}{(x - 3)(x + 3)} \\ \frac{\cancel{(x - 3)}(x + 2)}{\cancel{(x - 3)}(x + 3)} &= \frac{x + 2}{x + 3}\end{aligned}$$

Simplify $\frac{3x + 6}{x^2 + 5x + 6}$

Simplify $\frac{3x + 6}{x^2 + 5x + 6}$

The numerator has a common factor of 3; the denominator factorises into two brackets.

$$\begin{aligned}\frac{3x + 6}{x^2 + 5x + 6} &= \frac{3(x + 2)}{(x + 2)(x + 3)} \\ \frac{\cancel{3(x + 2)}}{\cancel{(x + 2)}(x + 3)} &= \frac{3}{x + 3}\end{aligned}$$

Simplify $\frac{2x^2 + 7x + 3}{x + 3}$

Simplify $\frac{2x^2 + 7x + 3}{x + 3}$

The coefficient of x^2 is 2, so one bracket starts with $2x$.

$$\frac{(2x + 1)\cancel{(x + 3)}}{\cancel{(x + 3)}} = 2x + 1$$

	$2x$	1
x	$2x^2$	x
3	$6x$	3

Simplify $\frac{2x^2 - 8}{x^2 + 4x + 4}$

Simplify $\frac{2x^2 - 8}{x^2 + 4x + 4}$

Take the common factor of 2 out of the numerator first.

$$\begin{aligned}\frac{2x^2 - 8}{x^2 + 4x + 4} &= \frac{2(x^2 - 4)}{(x + 2)(x + 2)} \\ \frac{2(x^2 - 4)}{(x + 2)(x + 2)} &= \frac{2(x - 2)\cancel{(x + 2)}}{(x + 2)\cancel{(x + 2)}} \\ \frac{2x^2 - 8}{x^2 + 4x + 4} &= \frac{2(x - 2)}{x + 2}\end{aligned}$$

Simplify $\frac{6x^2 + x - 2}{4x^2 - 1}$

Simplify $\frac{6x^2 + x - 2}{4x^2 - 1}$

The denominator is a difference of two squares.

$$6x^2 + x - 2 = (3x + 2)(2x - 1)$$

$$4x^2 - 1 = (2x - 1)(2x + 1)$$

$$\frac{(3x + 2)\cancel{(2x - 1)}}{(2x + 1)\cancel{(2x - 1)}} = \frac{3x + 2}{2x + 1}$$

Adding and subtracting algebraic fractions

True or false?

$$\frac{2}{x} + \frac{3}{x} = \frac{5}{2x}$$

True or false?

$$\frac{2}{x} + \frac{3}{x} = \frac{5}{2x}$$

False. The denominators are already the same, so only the numerators are added.

$$\frac{2}{x} + \frac{3}{x} = \frac{5}{x}$$

True or false?

$$\frac{x}{3} + \frac{x}{4} = \frac{7x}{12}$$

True or false?

$$\frac{x}{3} + \frac{x}{4} = \frac{7x}{12}$$

True. The common denominator is 12.

$$\frac{4x}{12} + \frac{3x}{12} = \frac{7x}{12}$$

True or false?

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$$

True or false?

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$$

False. Test it with $x = 2$ and $y = 3$: the left side is $\frac{5}{6}$ and the right side is $\frac{1}{5}$.

The common denominator is xy , so the answer is $\frac{x+y}{xy}$.

True or false?

$$\frac{4}{x} - \frac{x-3}{x} = \frac{4-x-3}{x}$$

True or false?

$$\frac{4}{x} - \frac{x-3}{x} = \frac{4-x-3}{x}$$

False. The minus sign applies to the whole second numerator, so the -3 becomes $+3$.

$$\frac{4-(x-3)}{x} = \frac{4-x+3}{x} = \frac{7-x}{x}$$

True or false?

The common denominator of $\frac{1}{x+1}$ and $\frac{2}{x+3}$ is $(x+1)(x+3)$

True or false?

The common denominator of $\frac{1}{x+1}$ and $\frac{2}{x+3}$ is
 $(x+1)(x+3)$

True. The denominators are multiplied, not added.

The common denominator is $(x+1)(x+3)$, not $x+4$.

Write as a single fraction

$$\frac{3}{x} + \frac{4}{x}$$

Write as a single fraction

$$\frac{3}{x} + \frac{4}{x}$$

The denominators are the same, so the numerators are added.

$$\frac{3}{x} + \frac{4}{x} = \frac{7}{x}$$

Write as a single fraction

$$\frac{x}{2} + \frac{x}{5}$$

Write as a single fraction

$$\frac{x}{2} + \frac{x}{5}$$

The common denominator is 10.

$$\frac{x}{2} + \frac{x}{5} = \frac{5x}{10} + \frac{2x}{10}$$

$$\frac{5x}{10} + \frac{2x}{10} = \frac{7x}{10}$$

Write as a single fraction

$$\frac{x}{4} + \frac{3x}{8}$$

Write as a single fraction

$$\frac{x}{4} + \frac{3x}{8}$$

8 is a multiple of 4, so the common denominator is 8.

$$\frac{x}{4} + \frac{3x}{8} = \frac{2x}{8} + \frac{3x}{8}$$

$$\frac{2x}{8} + \frac{3x}{8} = \frac{5x}{8}$$

Write as a single fraction

$$\frac{2x}{5} - \frac{x}{3}$$

Write as a single fraction

$$\frac{2x}{5} - \frac{x}{3}$$

The common denominator is 15.

$$\frac{2x}{5} - \frac{x}{3} = \frac{6x}{15} - \frac{5x}{15}$$

$$\frac{6x}{15} - \frac{5x}{15} = \frac{x}{15}$$

Write as a single fraction

$$\frac{x+1}{2} + \frac{x+2}{3}$$

Write as a single fraction

$$\frac{x+1}{2} + \frac{x+2}{3}$$

The common denominator is 6. Each numerator is multiplied by the same factor as its denominator.

$$\begin{aligned}\frac{x+1}{2} + \frac{x+2}{3} &= \frac{3(x+1) + 2(x+2)}{6} \\ \frac{3x+3+2x+4}{6} &= \frac{5x+7}{6}\end{aligned}$$

Write as a single fraction

$$\frac{2x + 3}{4} - \frac{x - 1}{6}$$

Write as a single fraction

$$\frac{2x + 3}{4} - \frac{x - 1}{6}$$

The common denominator is 12. The minus sign applies to the whole second numerator.

$$\begin{aligned}\frac{2x + 3}{4} - \frac{x - 1}{6} &= \frac{3(2x + 3) - 2(x - 1)}{12} \\ \frac{6x + 9 - 2x + 2}{12} &= \frac{4x + 11}{12}\end{aligned}$$

Write as a single fraction

$$\frac{2}{x} + \frac{3}{y}$$

Write as a single fraction

$$\frac{2}{x} + \frac{3}{y}$$

The common denominator is xy .

$$\begin{aligned}\frac{2}{x} + \frac{3}{y} &= \frac{2y}{xy} + \frac{3x}{xy} \\ \frac{2y}{xy} + \frac{3x}{xy} &= \frac{3x + 2y}{xy}\end{aligned}$$

Write as a single fraction

$$\frac{1}{x+1} + \frac{2}{x+3}$$

Write as a single fraction

$$\frac{1}{x+1} + \frac{2}{x+3}$$

The common denominator is $(x+1)(x+3)$.

$$\begin{aligned}\frac{1}{x+1} + \frac{2}{x+3} &= \frac{(x+3) + 2(x+1)}{(x+1)(x+3)} \\ \frac{x+3+2x+2}{(x+1)(x+3)} &= \frac{3x+5}{(x+1)(x+3)}\end{aligned}$$

Write as a single fraction

$$\frac{3}{x-1} - \frac{2}{x+2}$$

Write as a single fraction

$$\frac{3}{x-1} - \frac{2}{x+2}$$

The common denominator is $(x-1)(x+2)$. The minus sign applies to the whole second numerator.

$$\begin{aligned}\frac{3}{x-1} - \frac{2}{x+2} &= \frac{3(x+2) - 2(x-1)}{(x-1)(x+2)} \\ \frac{3x+6-2x+2}{(x-1)(x+2)} &= \frac{x+8}{(x-1)(x+2)}\end{aligned}$$

Write as a single fraction

$$\frac{1}{x-2} + \frac{3}{x^2-4}$$

Write as a single fraction

$$\frac{1}{x-2} + \frac{3}{x^2-4}$$

Factorise the second denominator, then write both fractions over $(x-2)(x+2)$.

$$x^2 - 4 = (x-2)(x+2)$$

$$\frac{1}{x-2} = \frac{x+2}{(x-2)(x+2)}$$

$$\frac{1}{x-2} + \frac{3}{x^2-4} = \frac{x+2+3}{(x-2)(x+2)}$$

$$\frac{x+2+3}{(x-2)(x+2)} = \frac{x+5}{(x-2)(x+2)}$$

Multiplying and dividing algebraic fractions

True or false?

To work out $\frac{a}{b} \div \frac{c}{d}$ you turn the first fraction upside down

True or false?

To work out $\frac{a}{b} \div \frac{c}{d}$ you turn the first fraction upside down

False. It is the second fraction that is turned upside down, and the two fractions are then multiplied.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

True or false?

$\frac{x}{2} \times \frac{3}{5}$ can be worked out without finding a common denominator

True or false?

$\frac{x}{2} \times \frac{3}{5}$ can be worked out without finding a common denominator

True. A common denominator is needed for adding and subtracting, not for multiplying.

The numerators are multiplied and the denominators are multiplied, so the answer is $\frac{3x}{10}$.

True or false?

$$\frac{a}{b} \times \frac{c}{d} = \frac{a + c}{b + d}$$

True or false?

$$\frac{a}{b} \times \frac{c}{d} = \frac{a+c}{b+d}$$

False. Test it with $\frac{1}{2} \times \frac{1}{2}$: the left side is $\frac{1}{4}$ and the right side is $\frac{2}{4} = \frac{1}{2}$.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

True or false?

$$\frac{x}{4} \div 2 = \frac{x}{2}$$

True or false?

$$\frac{x}{4} \div 2 = \frac{x}{2}$$

False. The whole number 2 is the fraction $\frac{2}{1}$, so it is turned upside down and the two are multiplied.

$$\frac{x}{4} \div \frac{2}{1} = \frac{x}{4} \times \frac{1}{2} = \frac{x}{8}$$

When $x = 8$: $2 \div 2 = 1$, not 4.

True or false?

$$\frac{x+1}{4} \times \frac{8}{x+1} \text{ simplifies to } 2$$

True or false?

$$\frac{x+1}{4} \times \frac{8}{x+1} \text{ simplifies to } 2$$

True. Cancelling across a multiplication sign is allowed, because $x+1$ is a factor of a whole numerator and of a whole denominator.

$$\frac{\cancel{8(x+1)}}{\cancel{4(x+1)}} = \frac{8}{4} = 2$$

Simplify

$$\frac{x}{2} \times \frac{3}{x}$$

Simplify

$$\frac{x}{2} \times \frac{3}{x}$$

Multiply the numerators and multiply the denominators, then cancel the common factor x .

$$\frac{x}{2} \times \frac{3}{x} = \frac{3x}{2x}$$

$$\frac{\cancel{3x}}{\cancel{2x}} = \frac{3}{2}$$

Simplify

$$\frac{3x}{4} \times \frac{8}{9x}$$

Simplify

$$\frac{3x}{4} \times \frac{8}{9x}$$

$$\frac{3x}{4} \times \frac{8}{9x} = \frac{24x}{36x}$$

$$\frac{\cancel{24x}}{\cancel{36x}} = \frac{24}{36}$$

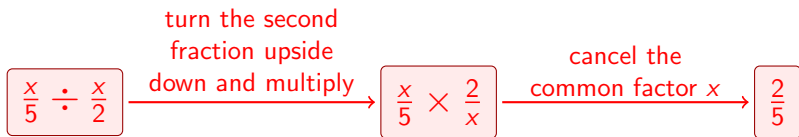
$$\frac{24}{36} = \frac{2}{3}$$

Simplify

$$\frac{x}{5} \div \frac{x}{2}$$

Simplify

$$\frac{x}{5} \div \frac{x}{2}$$

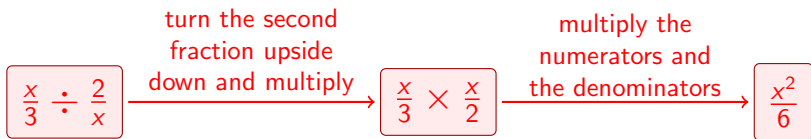


Simplify

$$\frac{x}{3} \div \frac{2}{x}$$

Simplify

$$\frac{x}{3} \div \frac{2}{x}$$



Simplify

$$\frac{x+1}{3} \times \frac{6}{x+2}$$

Simplify

$$\frac{x+1}{3} \times \frac{6}{x+2}$$

The two brackets are different, so only the numbers cancel.

$$\begin{aligned}\frac{x+1}{3} \times \frac{6}{x+2} &= \frac{6(x+1)}{3(x+2)} \\ \frac{6(x+1)}{3(x+2)} &= \frac{2(x+1)}{x+2}\end{aligned}$$

Simplify

$$\frac{x+2}{5} \div \frac{x+2}{10}$$

Simplify

$$\frac{x+2}{5} \div \frac{x+2}{10}$$

Turn the second fraction upside down and multiply.

$$\frac{x+2}{5} \div \frac{x+2}{10} = \frac{x+2}{5} \times \frac{10}{x+2}$$

$$\frac{x+2}{5} \times \frac{10}{x+2} = \frac{10(x+2)}{5(x+2)}$$

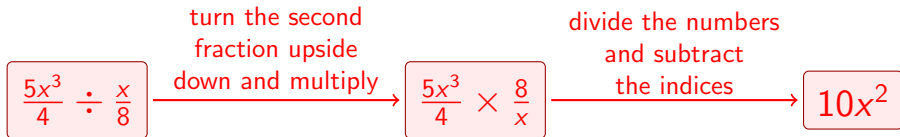
$$\frac{\cancel{10(x+2)}}{\cancel{5(x+2)}} = 2$$

Simplify

$$\frac{5x^3}{4} \div \frac{x}{8}$$

Simplify

$$\frac{5x^3}{4} \div \frac{x}{8}$$



Simplify

$$\frac{x^2 - 9}{x + 1} \times \frac{x + 1}{x - 3}$$

Simplify

$$\frac{x^2 - 9}{x + 1} \times \frac{x + 1}{x - 3}$$

$x^2 - 9$ is the difference of two squares.

$$x^2 - 9 = (x - 3)(x + 3)$$

$$\frac{x^2 - 9}{x + 1} \times \frac{x + 1}{x - 3} = \frac{(x - 3)(x + 3)(x + 1)}{(x + 1)(x - 3)}$$

$$\frac{\cancel{(x - 3)}(x + 3)\cancel{(x + 1)}}{\cancel{(x + 1)}\cancel{(x - 3)}} = x + 3$$

Simplify

$$\frac{x^2 + 7x + 10}{x - 1} \div \frac{x + 2}{x - 1}$$

Simplify

$$\frac{x^2 + 7x + 10}{x - 1} \div \frac{x + 2}{x - 1}$$

Factorise the quadratic, then turn the second fraction upside down and multiply.

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

$$\frac{(x + 2)(x + 5)}{x - 1} \div \frac{x + 2}{x - 1} = \frac{(x + 2)(x + 5)}{x - 1} \times \frac{x - 1}{x + 2}$$

$$\frac{\cancel{(x + 2)}(x + 5)\cancel{(x - 1)}}{\cancel{(x - 1)}\cancel{(x + 2)}} = x + 5$$

Simplify

$$\frac{x^2 + 3x + 2}{x^2 - 1} \div \frac{x^2 + 4x + 4}{x^2 - x}$$

Simplify

$$\frac{x^2 + 3x + 2}{x^2 - 1} \div \frac{x^2 + 4x + 4}{x^2 - x}$$

Factorise all four quadratics, then turn the second fraction upside down and multiply.

$$x^2 + 3x + 2 = (x + 1)(x + 2)$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$x^2 + 4x + 4 = (x + 2)(x + 2)$$

$$x^2 - x = x(x - 1)$$

$$\frac{(x + 1)(x + 2)}{(x + 1)(x - 1)} \times \frac{x(x - 1)}{(x + 2)(x + 2)} = \frac{(x + 1)(x + 2) \times (x - 1)}{(x + 1)(x - 1)(x + 2)(x + 2)}$$

$$\frac{\cancel{(x + 1)}\cancel{(x + 2)} \times \cancel{(x - 1)}}{\cancel{(x + 1)}\cancel{(x - 1)}\cancel{(x + 2)}(x + 2)} = \frac{x}{x + 2}$$